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Groups of obstructions to surgery and splitting for a manifold pair

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Abstract. The surgery obstruction groups LP_* of manifold pairs are studied. An algebraic version of these groups for squares of antistructures of a special form equipped with decorations is considered. The squares of antistructures in question are natural generalizations of squares of fundamental groups that occur in the splitting problem for a one-sided submanifold of codimension 1 in the case when the fundamental group of the submanifold is mapped epimorphically onto the fundamental group of the manifold. New connections between the groups LP_*, the Novikov–Wall groups, and the splitting obstruction groups are established.

Bibliography: 19 titles.

§ 1. Introduction

Let $f\colon M \to Y$ be a normal map of degree one of smooth (piecewise linear, topological) manifolds of dimension $n+q$, and let $X \subset Y$ be a submanifold of dimension n. Then the groups of obstructions to surgery $LP_n(F)$ for the manifold pair are defined. These groups depend functorially on the push-out square F of fundamental groups with orientation

$$\begin{array}{ccc} \pi_1(\partial U) & \longrightarrow & \pi_1(Y\setminus X) \\ \downarrow & & \downarrow \\ \pi_1(X) & \longrightarrow & \pi_1(Y), \end{array} \tag{1.1}$$

where U is a tubular neighbourhood of X and all the maps of fundamental groups are induced by the natural inclusions (see [1] and [2]). The groups $LP_n(F)$ are independent of the category of manifolds (smooth, piecewise linear, topological), as are the Novikov–Wall groups and almost all the natural maps considered in this paper. For this reason we shall use the piecewise linear vocabulary in what follows, pointing out distinctions from other categories when necessary. For $n \geqslant 5$ we have the obstruction $\sigma(f, M) \in LP_n(F)$, which is trivial if and only if there exists a map g such that g is transversal to X, lies in the class of the normal

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cobordism of f, $g^{-1}(X) = N$, and $g\colon (M; N, M \setminus N) \to (Y; X, Y \setminus X)$ is a simple homotopy equivalence of triples of spaces. If $n \leqslant 4$, then the condition $\sigma(f, M) = 0$ is necessary but not sufficient for the existence of a simple homotopy equivalence of triples.

Let $f\colon M \to Y$ be a normal map that is already a simple homotopy equivalence (in this case f is called a homotopy triangulation). Then there arises a problem of splitting along the submanifold X, with obstruction groups $LS_n(F)$. In this case the obstruction to splitting $\theta(f, M) \in LS_n(F)$ is zero if (and only if, for $n \geqslant 5$) there exists a simple homotopy equivalence of triples g lying in the homotopy class of the map f, transversal to X, and such that $g^{-1}(X) = N$ (see [1] and [2]).

The groups LS_* and LP_* are closely related to the surgery obstruction groups $L_*(\pi_1(Y))$ and $L_*(\pi_1(X))$, where π_1 is the fundamental group of the corresponding manifold. These groups occur naturally in many geometric problems. Apparently, the deepest relation between these spaces can be described by the following Levine braid of exact sequences (see [1]):

$$
\begin{array}{ccccccccccc}
\longrightarrow & L_{n+q}(C) & & \longrightarrow & L_{n+q}(D) & & \longrightarrow & & LS_{n-1}(F) & \longrightarrow & \\
& \nearrow \quad \searrow & & & \nearrow \quad \searrow & & & & \nearrow \quad \searrow & & \\
& & LP_n(F) & & & & L_{n+q}(C \to D) & & & & \\
& \searrow \quad \nearrow & & & \searrow \quad \nearrow & & & & \searrow \quad \nearrow & & \\
\longrightarrow & LS_n(F) & & \longrightarrow & L_n(B) & & \longrightarrow & & L_{n+q-1}(C) & \longrightarrow, &
\end{array}
\tag{1.2}
$$

where $A = \pi_1(\partial U)$, $B = \pi_1(X)$, $C = \pi_1(Y \setminus X)$, and $D = \pi_1(Y)$. The diagram (1.2) was constructed by Wall as an implement for calculating the L-groups and the natural maps involved. Later on, however, this diagram proved to be very efficient in geometric problems as well.

Let $hT(Y)$ be the set of equivalence classes of homotopy triangulations of a manifold Y. Two homotopy triangulations $f_i\colon M_i \to Y$ are equivalent if there exists a piecewise linear homeomorphism $h\colon M_1 \to M_2$ such that the maps f_2 and $f_1 h$ are homotopic. Each element (f, M) of the set $hT(Y)$ defines an obstruction to splitting $\theta(f, M) \in LS_n(F)$. Hence there exists a natural map $hT(Y) \to LS_n(F)$. We also consider the action λ of the group $L_{n+q+1}(\pi_1(Y))$ on the set $hT(Y)$. This is a map in the Sullivan exact sequence

$$
L_{n+q+1}(\pi_1(Y)) \xrightarrow{\lambda} hT(Y) \longrightarrow [Y, G/PL] \longrightarrow L_{n+q}(\pi_1(Y))\,. \tag{1.3}
$$

We now define a composite map

$$
r\colon L_{n+q+1}(\pi_1(Y)) \to hT(Y) \to LS_n(F),
$$

using the action λ of the group on the trivial triangulation.

If X is a one-sided submanifold of codimension 1 and the horizontal maps in the square F are isomorphisms (such pairs of manifolds are called Browder–Livesay pairs), then the splitting obstruction groups $LS_n(F)$ are the Browder–Livesay groups $LN_n(\pi_1(Y \setminus X) \to \pi_1(Y))$ (see [3] and [4]). In this case we have also an isomorphism $LP_n(F) \cong L_{n+1}(i^!)$, where $i^!\colon L_n(\pi_1(X)) \to L_n(\pi_1(\partial U))$ is the transfer map, and the map r is called the Browder–Livesay invariant (see [5]).

This map is a group homomorphism in the diagram (1.2). If $r(x) \neq 0$ for an element $x \in L_{n+q+1}(\pi_1(Y))$, then it follows from our description that the action of x on the set $hT(Y)$ is non-trivial. If $r(x) \neq 0$ then the element x cannot be realized by a normal map of closed manifolds according to [5].

We can regard the groups $LP_n(F)$ as 'dual' in a certain sense to the $LS_n(F)$, since (1.2) includes the long exact sequence

$$\longrightarrow LP_{n+1} \longrightarrow L_{n+q+1}(\pi_1(Y)) \longrightarrow LS_n(F) \longrightarrow .$$

The concept of the Browder–Livesay invariant has been developed in two directions. Based on the diagram (1.2) for the Browder–Livesay pairs, Kharshiladze [6] defined iterated Browder–Livesay invariants, which proved sufficient for the solution of the realization problem in the case of elementary Abelian 2-groups with an arbitrary orientation (see [6]). However, already in the case of finite 2-groups, the definitive results cannot be obtained using these methods (see [7]). There exists another approach to the realization problem (see [8]) which has allowed one to obtain more complete results in the oriented case.

On the other hand, a stronger (than r) generalized Browder–Livesay invariant was algebraically defined in [9] and [10]. This invariant is the homomorphism $L_{n+1}(\pi_1(Y)) \to LS_{n-1}(F)$ for a one-sided submanifold in the case when the horizontal maps in the square F are epimorphisms. Following [9], we call such a square a *geometric diagram*. Deep relations between the groups $LS_n(F)$, the Wall groups, and the Browder–Livesay groups were established for this case in [11] (see also [12]). These relations enable one, in particular, to generalize the groups $LS_*(F)$ for geometric diagrams F to the case of squares of antistructures equipped with decorations (see [11]).

The aim of this paper is to investigate the groups LP_* of obstructions to surgery for pairs of manifolds and to construct an algebraic version of these groups for *geometric diagrams of antistructures equipped with decorations*, which are natural generalizations of geometric diagrams of groups (see [11]). We establish here new relations between the groups LP_*, the Novikov–Wall groups, and the splitting obstruction groups. We must point out that the groups LP with decorations defined here are compatible with the earlier defined groups LS equipped with decorations (see [11]), so that we have diagram (1.2), also equipped with decorations this time (Theorem 4.2).

Several results of § 3 were announced in [12]. At the beginning of each section we describe briefly its contents.

§ 2. LP_* groups for a Browder–Livesay pair

We consider a Browder–Livesay pair of manifolds. That is, we assume that we have a one-sided submanifold of codimension 1 and that the horizontal maps in the square F are isomorphisms. In this case we have an algebraic version of diagram (1.2) constructed by Ranicki in [13] for the quadratic extension of antistructures (see also [14]). In [15], there is a description of the conditions on decorations ensuring that there exists a corresponding diagram of Novikov–Wall groups (in particular, of relative L-groups) equipped with decorations.

In this section we mainly consider a Browder–Livesay pair and give preliminary algebraic material concerning the groups LP_* of obstructions to surgery for pairs of manifolds and the splitting problem for this pair. Since the groups LP_* coincide in this case with the relative groups for the transfer map (see below), we can in a natural way introduce the relative groups for the LP_*. Using the methods of [15] we prove Theorem 2.1, which describes the diagram of relative groups of obstructions to surgery for pairs of manifolds.

In this paper, we use the concepts of an *antistructure* (R, α, u), a *quadratic extension of antistructures*, and of the decorated Novikov–Wall groups $L_n^X(R, \alpha, u)$. The necessary definitions are available in [11], [13], [15], and [16]. We shall simply write $L_n(R)$ if the type of antistructure under consideration is clear from the context. If a subgroup $X \subset K_1(R)$ is a decoration, then we assume that it is invariant with respect to the involution induced by α. We recall that for two decorations X and Y, $X \subset Y \subset K_1(R)$, we have the Rothenberg exact sequence (see [16])

$$\longrightarrow L_n^X(R) \longrightarrow L_n^Y(R) \longrightarrow H^n(Y/X) \longrightarrow, \tag{2.1}$$

where H^* is the Tate cohomology.

Let $i\colon (R, \alpha, u) \to (S, \alpha, u)$ be a quadratic extension of antistructures, and let (ρ, a) be a *structure* on R (see [13]). Let γ be the automorphism of the ring S over R described by the formula $\gamma(x + yt) = (x - yt)$, $x, y \in R$. The automorphism ρ extends to the ring S according to the formula $\rho(x + yt) = t(x + yt)t^{-1}$, $x, y \in R$. We have another antistructure, $(S, \widetilde{\alpha}, \widetilde{u})$, on S, where $\widetilde{\alpha} = \rho\gamma\alpha$ and $\widetilde{u} = -t\alpha(t^{-1})$. Since $\widetilde{u} \in R$ and the ring R is $\widetilde{\alpha}$-invariant, the antistructure $(R, \widetilde{\alpha}, \widetilde{u})$ is well defined. The quadratic extension of antistructures $(R, \widetilde{\alpha}, \widetilde{u}) \to (S, \widetilde{\alpha}, \widetilde{u})$ coincides with i as a ring inclusion.

Let $X \subset K_1(R)$ and $Y \subset K_1(S)$ be α- and $\widetilde{\alpha}$-invariant subgroups such that

$$i_*(X) \subset Y, \qquad i^!(Y) \subset X, \tag{2.2}$$

where i_* is the induced map and $i^!$ is the transfer map. We set $\widetilde{R} = (R, \widetilde{\alpha}, \widetilde{u})$ and $\widetilde{S} = (S, \widetilde{\alpha}, \widetilde{u})$; let $i^!_-\colon L_n^Y(S, \gamma\alpha, u) \to L_n^X(R, \alpha, u)$ be the transfer map, and let $i_*\colon L_n^X(R) \to L_n^Y(S)$ be the induced map (see [13]).

Then (see [13]–[15]) we have the Levine braid

$$\begin{array}{ccccccccccc}
\longrightarrow & & L_n^X(R) & & \xrightarrow{i_*} & & L_n^Y(S) & & \xrightarrow{i^!t^{-1}} & & L_{n-2}^X(\widetilde{R}) & & \xrightarrow{ti_*} \\
& \nearrow & & \searrow & & \nearrow & & \searrow & & \nearrow & & \searrow & \\
& & & & L_n^{Y,X}(i^!_-) & & & & L_n^{X,Y}(i_*) & & & & \\
& \searrow & & \nearrow & & \searrow & & \nearrow & & \searrow & & \nearrow & \\
\to & & L_{n-1}^X(\widetilde{R}) & & \xrightarrow{ti_*} & & L_{n-1}^Y(\widetilde{S}) & & \xrightarrow{i^!} & & L_{n-1}^X(R) & & \longrightarrow,
\end{array} \tag{2.3}$$

where the $L_n^{Y,X}(i^!_-)$ and $L_n^{X,Y}(i_*)$ are the relative groups (see [2]).

We now consider the inclusion $i\colon \pi \to G$ of index 2 of groups with orientation that corresponds to the right vertical column in the square F related to the Browder–Livesay pair. Let $w\colon G \to \{\pm 1\}$ be the orientation homomorphism, and let $t \in G \backslash \pi$. Then

$$LN_{2n}(\pi \to G) \cong L_{2n}^U(\mathbb{Z}\pi, \alpha, -w(t)t^{-2}), \tag{2.4}$$

where $\alpha(x) = t\overline{x}t^{-1}$ is an anti-automorphism on the ring $\mathbb{Z}\pi$, and U is the subgroup of $K_1(\mathbb{Z}\pi)$ generated by the elements $\pm g$, $g \in \pi$ (see [17]). (We recall that the bar denotes the standard involution $\overline{\sum a_g g} = \sum w(g)a_g g^{-1}$.) In odd dimensions one must consider the quotient of the right-hand side of (2.4) modulo the subgroup $\mathbb{Z}/2$ generated by the automorphism $\begin{pmatrix} 0 & 1 \\ \pm w(t)t^{-2} & 0 \end{pmatrix}$. Diagram (1.2) is isomorphic in this case to diagram (2.3) defined for the quadratic extension $i\colon (\mathbb{Z}\pi, -, 1) \to (\mathbb{Z}G, -, 1)$ of antistructures equipped with the decorations U (see [18]). In particular, we have the isomorphism

$$LP_n(F) \cong L_{n+1}(i^!_-),$$

where $i^!_-\colon L_n(\mathbb{Z}G, -, 1) \to L_n(\mathbb{Z}\pi, -, 1)$.

Thus, if $i\colon (R, \alpha, u) \to (S, \alpha, u)$ is a quadratic extension of antistructures and if conditions (2.2) on decorations are satisfied, then the groups $L^{Y,X}_n(i^!_-)$ are natural generalizations of the groups of obstructions to surgery for manifold pairs in the Browder–Livesay case. We denote these groups by $LP^{Y,X}_{n-1}(\Phi)$. Here Φ is a commutative square of antistructures in which the vertical maps are the extension i and the horizontal maps are isomorphisms. We do not specify the decorations in question if they are clear from the context. We recall (see [13] and [14]) that we have the isomorphism $L^{Y,X}_n(i^!_-) \cong L^{Y,X}_{n-1}(\tilde{i}^!_-)$, where $\tilde{i}^!_-$ is the transfer map

$$\tilde{i}^!_-\colon (S, \gamma\widetilde{\alpha}, \widetilde{u}) \to (R, \widetilde{\alpha}, \widetilde{u}).$$

We now consider a commutative square Ψ of antistructures

$$\begin{array}{ccc} (R, \alpha, u) & \xrightarrow{f} & (R, \beta, v) \\ \downarrow{\scriptstyle i} & & \downarrow{\scriptstyle j} \\ (S, \alpha, u) & \xrightarrow{g} & (Q, \beta, v), \end{array} \tag{2.5}$$

which defines a quadratic extension g of the morphism f of antistructures (see [15]).

Assume that the subgroups $X \subset K_1(R)$, $Z \subset K_1(P)$, $Y \subset K_1(S)$, $W \subset K_1(Q)$ are invariant with respect to the two corresponding involutions and

$$\begin{array}{lll} i_*(X) \subset Y, & j_*(Z) \subset W, & f_*(X) \subset Z, \\ g_*(Y) \subset W, & i^!(Y) \subset X, & j^!(W) \subset Z. \end{array} \tag{2.6}$$

Let Φ' be a square of antistructures containing two isomorphic vertical morphisms j. Under these assumptions (see [2], [13]), there exist naturally defined relative groups $LP_*(f, g)$ fitting into the exact sequence

$$\longrightarrow LP^{Y,X}_n(\Phi) \longrightarrow LP^{W,Z}_n(\Phi') \longrightarrow LP_n(f, g) \longrightarrow . \tag{2.7}$$

It follows from [15] that if conditions (2.6) are satisfied, then we have diagram (2.3) for the relative L-groups of the horizontal maps in diagram (2.5). This diagram involves also the groups $LP_*(f, g)$.

Remark. We can define the groups $LP_*(f,g)$ in a natural way also as the relative groups of the transfer map $i^!_{\mathrm{rel}}$ of the relative groups

$$L^{Y,W}_*\big((S,\gamma\alpha,u)\to(Q,\gamma\beta,v)\big)\longrightarrow L^{X,Z}_*\big((R,\alpha,u)\to(P,\beta,v)\big),$$

This follows easily from the commutative diagram for the transfer maps in (2.5).

Assume that the horizontal maps in the square (2.5) also define a quadratic extension j of i. Let $\tau\in P\subset Q$ be an element generating these extensions. In a similar way to the construction of the morphism of antistructures 'with tildes' for the quadratic extension i, we can construct the commutative square of antistructures

$$\begin{array}{ccc} (R,\overline{\alpha},\overline{u}) & \xrightarrow{\overline{f}} & (P,\overline{\beta},\overline{v}) \\ \downarrow{\scriptstyle\overline{i}} & & \downarrow{\scriptstyle\overline{j}} \\ (S,\overline{\alpha},\overline{u}) & \xrightarrow{\overline{g}} & (Q,\overline{\beta},\overline{v}), \end{array} \tag{2.8}$$

where the vertical and the horizontal maps are quadratic extensions again. Let $\overline{\Phi}$ be the square of antistructures in which the two vertical columns are the same as the left-hand column in the square (2.8), and the horizontal maps are isomorphisms. In the same way, starting from the right-hand column in (2.8) we define a square $\overline{\Phi'}$.

Theorem 2.1. *Assume that the horizontal maps in the square* (2.5) *define a quadratic extension and that the decorations X, Z, Y, and W satisfy conditions* (2.6). *Then we have the following Levine braid*:

$$\begin{array}{ccccccccccc} \longrightarrow & LP^{Y,X}_n(\Phi) & & \xrightarrow{(f,g)_*} & & LP^{W,Z}_n(\Phi') & & \longrightarrow & & LP^{Y,X}_{n-2}(\overline{\Phi}) & \longrightarrow \\ & \nearrow & \searrow & & \nearrow & & \searrow & & \nearrow & \searrow & \\ & & & \Delta(f,g)^! & & & & \Delta(f,g)_* & & & \\ & \searrow & \nearrow & & \searrow & & \nearrow & & \searrow & \nearrow & \\ \longrightarrow & LP^{Y,X}_{n-1}(\overline{\Phi}) & & \xrightarrow{\tau(f,g)_*} & & LP^{W,Z}_{n-1}(\overline{\Phi'}) & & \longrightarrow & & LP^{Y,X}_{n-1}(\Phi) & \longrightarrow, \end{array} \tag{2.9}$$

where $\Delta(f,g)^!$ and $\Delta(f,g)_*$ are the corresponding relative groups.

Proof. We consider the diagram (2.3) for the quadratic extensions f and

$$g_\gamma\colon(S,\gamma\alpha,u)\to(Q,\gamma\beta,v),$$

where $\gamma(t)=-t$, $\gamma\big|_R$ and $\gamma\big|_P$ are the identity maps. Since these diagrams are natural, the vertical maps in (2.5) and (2.8) define the transfer map from the diagram for the quadratic extension g_γ into the diagram for f. From now on, we can repeat almost word for word the proof of Theorem 2 in [15]. All the maps in the indicated diagrams and the transfer map of the diagrams can be realized on the spectral level (see [19]). The cofibres of the transfer maps between the diagrams are spectra, the homotopy groups of these spectra give us diagram (2.9), and the sequences in this diagram are exact since the squares of the spectra are pull-backs (see [15]), which proves the theorem.

§ 3. Groups LP_* for geometric diagrams of groups

In this section we construct Levine braids connecting the groups LP_* with the Novikov–Wall groups for geometric diagrams of groups. We shall also define the groups LP_* for squares of antistructures (2.5) in which the horizontal maps are epimorphisms. Such squares are natural generalizations of geometric diagrams of groups, and the groups that we introduce keep all the algebraic properties of the surgery obstruction groups for manifold pairs.

We now denote by Ψ a commutative square of antistructures (2.5) in which the horizontal maps are epimorphisms and the morphism g of antistructures is a quadratic extension of the morphism f. We call a square of antistructures with such properties a *geometric diagram of antistructures* (see [11]). Using the square Ψ we can construct a geometric diagram Ψ_γ with morphism $g_\gamma\colon (S,\gamma\alpha,u)\to(Q,\gamma\beta,v)$ in the lower row, and with upper row and ring morphisms as in Ψ.

By [19] and [11] the square Ψ_γ generates the homotopy commutative square of Quinn–Ranicki spectra

$$\begin{array}{ccc} \mathbb{L}(R,\alpha,u) & \xrightarrow{\mathbb{L}(f)} & \mathbb{L}(P,\beta,v) \\ \uparrow{\scriptstyle \underline{i}^!_-} & & \uparrow{\scriptstyle \underline{j}^!_-} \\ \mathbb{L}(S,\gamma\alpha,u) & \xrightarrow{\mathbb{L}(g_\gamma)} & \mathbb{L}(Q,\gamma\beta,v). \end{array} \tag{3.1}$$

The homotopy groups of the spectra in this square are isomorphic to the corresponding L-groups. The horizontal maps are induced by f and g, and the vertical maps correspond to the transfer maps (see [19]). For spectra, we use the notation of [11] by setting $\Omega\mathbb{L}_{n+1}\cong\mathbb{L}_n$, denoting the homotopy cofibre of the map $\mathbb{L}(f)$ of the spectra by $\mathbb{L}(f_*)$, the homotopy cofibre of the map $\underline{i}^!$ by $\mathbb{L}(i^!)$, and setting $\pi_n(\mathbb{L}(*))=L_n(*)$.

The diagram (3.1) can be extended in all directions so that all the rows and columns are cofibrations and the diagram remains homotopy commutative (see [11]). The resulting diagram also involves the spectrum $\mathbb{L}(\Psi^!_\gamma)$, which is the homotopy cofibre of the map $\mathbb{L}(i^!)\to\mathbb{L}(j^!)$. We now consider the homotopy commutative square of this extended diagram:

$$\begin{array}{ccc} \Omega\mathbb{L}(\Psi^!_\gamma) & \longrightarrow & \mathbb{L}(i^!) \\ \uparrow & & \uparrow \\ \Omega\mathbb{L}(f_*) & \longrightarrow & \mathbb{L}(R,\alpha,u) \end{array} \tag{3.2}$$

and we define the spectrum $\mathbb{LP}(\Psi)$ as the homotopy fibre of the diagonal map in the square (3.2)).

Before describing the connections of this spectrum with other L-spectra and studying its homotopy groups we consider the square Ψ of antistructures that corresponds to the geometric diagram of groups (1.1). We set $A=\pi_1(\partial U)$, $B=\pi_1(X)$, $C=\pi_1(Y\setminus X)$, and $D=\pi_1(Y)$. We can construct the geometric diagram of antistructures from the square F by passing to the group rings over $\mathbb{Z}$. The involutions in all the rings except for $\mathbb{Z}B$ correspond in that case to the standard involution defined by the orientation of the manifold. We choose the involution in the ring $\mathbb{Z}B$

compatible with the ring homomorphisms (see [11]). The invertible element u is equal to 1. Hence we obtain the following geometric diagram $\mathbb{Z}F$ of antistructures:

$$\begin{array}{ccc} (\mathbb{Z}A, \text{-}, 1) & \longrightarrow & (\mathbb{Z}C, \text{-}, 1) \\ \downarrow & & \downarrow \\ (\mathbb{Z}B, \text{-}, 1) & \longrightarrow & (\mathbb{Z}D, \text{-}, 1), \end{array} \tag{3.3}$$

where the orientation homomorphism of group B on the left-hand side does not correspond to the orientation of the submanifold X in (1.1).

Theorem 3.1. *Let F be a geometric diagram of groups and let $\mathbb{Z}F$ be the geometric diagram of antistructures* (3.2) *constructed from the square F as described above. Then we have the isomorphism*

$$\pi_n(\mathbb{LP}(\mathbb{Z}F)) \cong LP_n(F),$$

where the $LP_n(F)$ are the surgery obstruction groups on the pair of manifolds corresponding to the square F.

Proof. We consider a natural map Λ of manifold pairs inducing a natural map of the diagram Φ, which is the diagram

$$\begin{array}{ccc} A & \longrightarrow & A \\ \downarrow & & \downarrow \\ B & \longrightarrow & B, \end{array}$$

into the diagram F. Such a map of manifold pairs exists by [1] and [2]. Since the Levine braid (1.2) is natural, we obtain the map (1.2) of the diagrams induced by Λ, where the maps of the Novikov–Wall groups and the splitting obstruction groups are induced by the maps of the squares. Since the diagram (1.2) can be realized on the spectral level (see, for example, [18]), the map Λ induces a map of the commutative squares of spectra:

$$\begin{array}{ccc} \Omega\mathbb{L}(i^!_-) & \longrightarrow & \Omega\mathbb{L}(B) \\ \downarrow & & \downarrow \\ \mathbb{L}(B^-) & \longrightarrow & \Omega\mathbb{L}(A \to B) \end{array} \xrightarrow{\mathbb{L}(\Lambda)} \begin{array}{ccc} \mathbb{LP}(F) & \longrightarrow & \Omega\mathbb{L}(D) \\ \downarrow & & \downarrow \\ \mathbb{L}(B^-) & \longrightarrow & \Omega\mathbb{L}(C \to D), \end{array} \tag{3.4}$$

where the orientation homomorphism of the group B corresponds to the orientation of the manifold Y, and the orientation homomorphism of the group B^- corresponds to the orientation of the submanifold X. We now consider the universal square of the fibres of $\mathbb{L}(\Lambda)$, which is

$$\begin{array}{ccc} \Omega^2\mathbb{L}(A \to C) & \longrightarrow & \Omega^2\mathbb{L}(B \to D) \\ \downarrow & & \downarrow \\ * & \longrightarrow & \Omega^2\mathbb{L}(F)\,, \end{array}$$

since the maps on the Novikov–Wall groups are natural and the squares of spectra are universal. In particular, we obtain the fibration

$$\Omega^2\mathbb{L}(A \to C) \longrightarrow \Omega\mathbb{L}(i^!_-) \longrightarrow \mathbb{LP}(F).$$

The first map in this fibration is the natural composite

$$\Omega^2\mathbb{L}(A \to C) \longrightarrow \Omega\mathbb{L}(A) \longrightarrow \Omega\mathbb{L}(i^!_-)$$

of a map in the relative exact sequence for the map $A \to C$ and of a map in the relative exact sequence for the transfer map $i^!_- \colon L_*(B^-) \to L_*(A)$ considered each on the spectral level (see [19]). This is a consequence of the universality of the left-hand square in diagram (2.4) and of the fact that the maps $\Omega^2\mathbb{L}(A \to C) \longrightarrow \mathbb{L}(B^-)$ and $\Omega^2\mathbb{L}(A \to C) \longrightarrow \Omega\mathbb{L}(B)$ factor through the natural map $\Omega^2\mathbb{L}(A \to C) \longrightarrow \Omega\mathbb{L}(A)$. The theorem now follows from the definition of the spectrum $\mathbb{LP}(\mathbb{Z}F)$ and diagrams (3.2) and (3.3).

Theorem 3.2. *Consider the following maps of spectra, which are the composites of the natural maps in diagram* (3.1) *extended by cofibrations*:

$$\Omega\mathbb{L}(f_*) \longrightarrow \mathbb{L}(i^!), \quad \mathbb{L}(S, \gamma\alpha, u) \xrightarrow{\delta} \mathbb{L}(P, \beta, v), \quad \Omega\mathbb{L}(j^!) \longrightarrow \mathbb{L}((g_\gamma)_*).$$

The spectrum $\mathbb{LP}(\Psi)$, which was defined above as the homotopy fibre of the first of these maps, is homotopy equivalent to the fibre of any of these maps of spectra.

Proof. In the category of spectra, fibrations coincide with cofibrations. It is now sufficient to consider diagram (3.1) extended by cofibrations and to apply Lemma 2 in [11].

For a geometric diagram Ψ of antistructures let $LP_n(\Psi)$ be the homotopy groups $\pi_n(\mathbb{LP}(\Psi))$. This definition is consistent by Theorem 3.1

Corollary 3.1. *If Ψ is a geometric diagram of antistructures, then we have the following long exact sequences*:

$$\begin{gathered} \longrightarrow L_{n+1}(f) \longrightarrow L_n(i^!_-) \longrightarrow LP_{n-1}(\Psi) \longrightarrow, \\ \longrightarrow L_n(S, \gamma\alpha, u) \longrightarrow L_n(P, \beta, v) \longrightarrow LP_{n-1}(\Psi) \longrightarrow, \\ \longrightarrow L_{n+1}(j^!_-) \longrightarrow L_n(g_\gamma) \longrightarrow LP_{n-1}(\Psi) \longrightarrow, \end{gathered} \tag{3.5}$$

in which the maps $i^!_-, j^!_-$ are the transfer maps for the diagram Ψ_γ.

Proof. These exact sequences are the homotopy long exact sequences of the fibrations in Theorem 3.2.

Now let F be a geometric diagram

$$\begin{array}{ccc} A & \longrightarrow & C \\ \downarrow & & \downarrow \\ B^- & \longrightarrow & D^+ \end{array}$$

of groups equipped with orientations. Then we can define geometric diagrams Φ and Φ' of groups as follows:

$$\begin{array}{ccc} A & \longrightarrow & A \\ \downarrow & & \downarrow \\ B^- & \longrightarrow & B^+ \end{array} \quad \text{and} \quad \begin{array}{ccc} C & \longrightarrow & C \\ \downarrow & & \downarrow \\ D^- & \longrightarrow & D^+. \end{array}$$

These diagrams correspond to Browder–Livesay pairs. We also have natural maps $\Phi \to F \to \Phi'$ of these diagrams. We can now write the exact sequences from Corollary 3.1 in a form convenient for geometric applications.

Corollary 3.2. *For the diagrams F, Φ, and Φ' defined above we have the following long exact sequences*:

$$\longrightarrow L_{n+1}(A\to C)\longrightarrow LP_{n-1}(\Phi)\longrightarrow LP_{n-1}(F)\longrightarrow,$$

$$\longrightarrow L_n(B^-)\longrightarrow L_n(C)\longrightarrow LP_{n-1}(F)\longrightarrow,$$

$$\longrightarrow LP_n(\Phi')\longrightarrow L_n(g_-)\longrightarrow LP_{n-1}(F)\longrightarrow,$$

in which the orientation on the group B^- corresponds to the orientation of the submanifold, and $g_-\colon B^-\to D^-$ is a map of groups with orientation.

We must point out that g_- is not involved in the geometric diagrams F, Φ, and Φ'.

Theorem 3.3 and Corollary 3.3 to it, which follow, are counterparts of Theorem 4 and Corollary 2 in [11]. The proofs of these results are also perfectly similar to those in [11].

Theorem 3.3. *Let Ψ be a geometric diagram of antistructures. Then we have the following universal squares of spectra*

$$\begin{array}{ccc} \Omega\mathbb{L}(R) & \longrightarrow & \Omega\mathbb{L}(P)\\ \downarrow & & \downarrow\\ \Omega\mathbb{L}(i^!_-) & \longrightarrow & \mathbb{LP}(\Psi), \end{array}$$

$$\begin{array}{ccc} \Omega\mathbb{L}(Q,\gamma\alpha,u) & \longrightarrow & \Omega\mathbb{L}((g_\gamma)_*)\\ \downarrow & & \downarrow\\ \Omega\mathbb{L}(P) & \longrightarrow & \mathbb{LP}(\Psi), \end{array}$$

$$\begin{array}{ccc} \Omega^2\mathbb{L}(\Psi^!_\gamma) & \longrightarrow & \Omega\mathbb{L}(i^!_-)\\ \downarrow & & \downarrow\\ \Omega\mathbb{L}((g_\gamma)_*) & \longrightarrow & \mathbb{LP}(\Psi), \end{array}$$

in which the transfer maps are as in Corollary 3.1.

Corollary 3.3. *Under the hypothesis of Theorem* 3.3 *we have the following Levine braids*:

$$\begin{array}{ccccccccccc}
\longrightarrow & L_{n+1}(S,\gamma\alpha,u) & & \longrightarrow & & L_{n+1}(P) & & \longrightarrow & & L_{n+1}(f) & \longrightarrow\\
& \nearrow & \searrow & & \nearrow & & \searrow & & \nearrow & \searrow & \\
& & & L_{n+1}(R) & & & & LP_n(\Psi) & & & \\
& \searrow & \nearrow & & \searrow & & \nearrow & & \searrow & \nearrow & \\
\longrightarrow & L_{n+2}(f) & & \longrightarrow & & L_{n+1}(i^!_-) & & \longrightarrow & & L_n(S,\gamma\alpha,u) & \longrightarrow,
\end{array}$$

$$\begin{array}{ccccccccccc}
\longrightarrow & L_{n+2}(j^!_-) & & \longrightarrow & & L_{n+1}(g_\gamma) & & \longrightarrow & & L_n(S,\gamma\alpha,u) & \longrightarrow\\
& \nearrow & \searrow & & \nearrow & & \searrow & & \nearrow & \searrow & \\
& & & L_{n+1}(Q,\gamma\alpha,u) & & & & LP_n(\Psi) & & & \\
& \searrow & \nearrow & & \searrow & & \nearrow & & \searrow & \nearrow & \\
\longrightarrow & L_{n+1}(S,\gamma\alpha,u) & & \longrightarrow & & L_{n+1}(P) & & \longrightarrow & & L_{n+1}(j^!_-) & \longrightarrow,
\end{array} \tag{3.6}$$

$$
\begin{tikzcd}[column sep=small]
{} \arrow[r] & L_{n+2}(f) \arrow[rr] \arrow[dr] & & L_{n+1}(i^!_-) \arrow[rr] \arrow[dr] & & L_{n+1}(j^!_-) \arrow[r] \arrow[dr] & {} \\
{} \arrow[ur] \arrow[dr] & & L_{n+2}(\Psi^!_\gamma) \arrow[ur] \arrow[dr] & & LP_n(\Psi) \arrow[ur] \arrow[dr] & & {} \\
{} \arrow[r] & L_{n+2}(j^!_-) \arrow[rr] \arrow[ur] & & L_{n+1}(g_\gamma) \arrow[rr] \arrow[ur] & & L_{n+1}(f) \arrow[r] \arrow[ur] & {}.
\end{tikzcd}
$$

We note that one encounters no difficulties in modifying Corollary 3.3 for the case of a geometric diagram of groups by analogy with Corollary 3.2.

§ 4. LP_*-groups with decorations

In this section we define the groups $LP^k_*(\Psi)$ for a geometric diagram of antistructures Ψ equipped with a square of decorations k satisfying some additional conditions. We prove that all the relations (such as Corollary 3.3) between the 'decorated surgery obstruction groups for pairs of manifolds' and the Novikov–Wall groups (now also with decorations) are as before. We shall study the dependence of the groups LP^k_* on decorations. This section is similar to § 3 in [11], where we define the decorated groups LS^k_* for geometric diagrams of antistructures. However, there are also important distinctions between this section and § 3 in [11], which stem mainly from using the square Ψ_γ in place of the square of antistructures 'with tildes' in the definitions of the corresponding groups.

We note that the decorated groups LP^k in this paper are compatible with the decorated groups LS^k (equipped with the same square of decorations) introduced in [11]. This enables us to construct diagram (1.2) for groups with decorations, which is very important in geometric applications.

We now consider a geometric diagram of antistructures Ψ (see (2.5)); let k be the commutative square

$$
\begin{array}{ccc}
X & \xrightarrow{f_*} & Z \\
\big\downarrow{\scriptstyle i_*} & & \big\downarrow{\scriptstyle j_*} \\
Y & \xrightarrow{g_*} & W
\end{array}
\tag{4.1}
$$

of the decorations defined in § 2, which are invariant with respect to the two corresponding involutions and satisfy conditions (2.6). All the maps in k are induced by the maps in Ψ, and the pair (Ψ, k) defines a *geometric diagram of antistructures* Ψ^k *equipped with decorations* (see [11]).

As before, we have in this case diagrams similar to (3.1) and (3.2), but now involving the spectra defining the Novikov–Wall groups equipped with decorations (see [19]). As in § 2, we define the spectrum $\mathbb{LP}(\Psi^k)$ as the homotopy fibre of the natural map of spectra

$$
\Omega\mathbb{L}(f_*) \longrightarrow \mathbb{L}(i^!_-),
$$

where $i^!_-\colon L^Y_n(S, \gamma\alpha, u) \to L^X_n(R, \alpha, u)$ is the transfer map of the decorated Novikov–Wall groups and $f_*\colon L^X_n(R, \alpha, u) \to L^Z_n(P, \beta, v)$ is the induced map. This definition is compatible with the definition of the LP-groups with decorations for Browder–Livesay pairs in § 2. The following theorem is perfectly similar to Theorem 6 in [11].

Theorem 4.1. *Theorems* 3.3 *and* 3.4*, where all the spectra are equipped with the corresponding decorations from the square* k*, hold for the spectrum* $\mathbb{LP}(\Psi^k)$*.*

Definition. We call the homotopy groups of the spectrum $\mathbb{LP}(\Psi^k)$ the *LP-groups with decorations k*.

We shall denote these groups by $LP^k(\Psi)$, by analogy with the Novikov–Wall groups and the LS-groups with decorations (see [11]).

Corollary 4.1. *For the groups $LP^k(\Psi)$ we have the exact sequences* (3.5) *and the Levine braids* (3.6)*, where all the groups are equipped with the corresponding decorations.*

In the following result we shall use the groups $LS^k_*(\Psi)$ defined in [11]. We note only that these groups are defined for the geometric diagram of antistructures Ψ^k equipped with decorations.

Theorem 4.2. *Let Ψ^k be the above-defined geometric diagram of antistructures equipped with decorations. Then there exists the following Levine braid*:

$$\begin{tikzcd}[column sep=small]
{} \arrow[r] & L^Z_{n+1}(P,\beta,v) \arrow[rr] \arrow[dr] & & L^W_{n+1}(Q,\beta,v) \arrow[rr] \arrow[dr] & & LS^k_{n-1}(\Psi) \arrow[r] \arrow[dr] & {} \\
{} \arrow[ur] \arrow[dr] & & LP^k_n(\Psi) \arrow[ur] \arrow[dr] & & L^{Z,W}_{n+1}(P \to Q,\beta,v) \arrow[ur] \arrow[dr] & & {} \\
{} \arrow[r] & LS^k_n(\Psi) \arrow[rr] \arrow[ur] & & L^Y_n(S,\gamma\alpha,u) \arrow[rr] \arrow[ur] & & L^Z_n(P,\beta,v) \arrow[r] \arrow[ur] & {} .
\end{tikzcd}$$

Proof. By [11] and [13] we have the cofibration of spectra

$$\Omega\mathbb{L}\big((S,\gamma\alpha,u)^Y\big) \xrightarrow{\varepsilon} \Omega^2\mathbb{L}\big((P \to Q,\beta,v)^{Z,W}\big) \longrightarrow \mathbb{LS}(\Psi^k), \tag{4.2}$$

in which the map ε is the composite of the natural maps of spectra

$$\Omega\mathbb{L}\big((S,\gamma\alpha,u)^Y\big) \longrightarrow \Omega\mathbb{L}\big((Q,\gamma\beta,v)^W\big) \longrightarrow \Omega\mathbb{L}\big((P \to Q,\widetilde{\beta},\widetilde{v})^{Z,W}\big)$$

and of the natural isomorphism

$$\Omega\mathbb{L}\big((P \to Q,\widetilde{\beta},\widetilde{v})^{Z,W}\big) \cong \Omega^2\mathbb{L}\big((P \to Q,\beta,v)^{Z,W}\big).$$

For convenience of notation we indicate here the decorations by superscripts at the corresponding antistructures. The map δ in Theorem 3.2 is the composite $\mathbb{L}(g_\gamma)\underline{j}^!_-$ of the maps involved in the square (3.1), which can also be defined for the geometric diagrams of antistructures equipped with decorations. In addition, we have the commutative diagram (see [13] and [15])

$$\begin{array}{ccc}
\mathbb{L}\big((Q,\gamma\beta,v)^W\big) & \xrightarrow{\underline{j}^!_-} & \mathbb{L}((P,\beta,v)^Z) \\
\big\downarrow & & \big\uparrow \\
\mathbb{L}\big((\widetilde{P} \to \widetilde{Q},\widetilde{\beta},\widetilde{v})^{Z,W}\big) & \longrightarrow & \Omega\mathbb{L}\big((P \to Q,\beta,v)^{Z,W}\big).
\end{array}$$

Hence we have the commutative square of spectra

$$\begin{array}{ccc}
\mathbb{L}\big((S,\gamma\alpha,u)^Y\big) & \xrightarrow{\delta} & \mathbb{L}\big((P,\beta,v)^Z\big) \\
\big\downarrow{\scriptstyle\varepsilon} & & \big\uparrow{\scriptstyle\cong} \\
\Omega\mathbb{L}\big((P \to Q,\beta,v)^{Z,W}\big) & \longrightarrow & \mathbb{L}\big((P,\beta,v)^Z\big).
\end{array}$$

We now consider the map $\nu\colon \mathbb{LP}(\Psi^k) \to \Omega\mathbb{L}\big((Q,\beta,v)^Z\big)$ of the fibres of the horizontal maps induced by this square, so that we obtain the commutative square

$$\begin{array}{ccc} \mathbb{LP}(\Psi^k) & \longrightarrow & \mathbb{L}\big((S,\gamma\alpha,u)^Y\big) \\ \downarrow{\scriptstyle \nu} & & \downarrow{\scriptstyle \varepsilon} \\ \Omega\mathbb{L}\big((Q,\beta,v)^W\big) & \longrightarrow & \Omega\mathbb{L}\big((P\to Q,\beta,v)^{Z,W}\big), \end{array} \tag{4.3}$$

in which the cofibres of the horizontal maps are naturally homotopy equivalent to the spectrum $\mathbb{L}((P,\beta,v)^Z)$. That is, we obtain a universal square of spectra. It now follows from the cofibration (4.2) that the fibres of the vertical maps are homotopy equivalent to the spectrum $\mathbb{LS}(\Psi^k)$. The homotopy long exact sequences for the maps in (4.3) form the required Levine braid, which completes the proof.

We study now the behaviour of the LP^k-groups under changes of decorations. We shall obtain certain analogues of the Rothenberg exact sequence (2.1) for this case.

First we consider the simpler case of a geometric antistructure Φ with decorations k defined in § 2. We recall that this diagram corresponds geometrically to a Browder–Livesay pair. Assume that we also have a square l of decorations such that we have an inclusion $k \subset l$ of the following form:

$$\begin{array}{ccc} X & \longrightarrow & X \\ \downarrow & & \downarrow \\ Y & \longrightarrow & Y \end{array} \subset \begin{array}{ccc} X' & \longrightarrow & X' \\ \downarrow & & \downarrow \\ Y' & \longrightarrow & Y', \end{array}$$

and let Φ^l be a geometric diagram of antistructures equipped with decorations.

Theorem 4.3. *Under the above assumptions we have the long exact sequence*

$$\longrightarrow LP^k_*(\Phi) \longrightarrow LP^l_*(\Phi) \longrightarrow H^{*+1}(i^!_-) \longrightarrow, \tag{4.4}$$

in which $H^{*+1}(i^!_-)$ denotes the relative Tate cohomology included into the long exact sequence

$$\longrightarrow H^*(Y'/Y) \xrightarrow{i^!_-} H^*(X'/X) \longrightarrow H^*(i^!_-) \longrightarrow .$$

Here the involution on the groups X and X' is induced by α, while the involution on the groups Y and Y' is induced by $\gamma\alpha$.

Proof. The Rothenberg exact sequence (2.1) for the antistructure $(S,\gamma\alpha,u)$ with decorations Y and Y' is mapped into the Rothenberg exact sequence for the antistructure (R,α,u) with decorations X and X', since the transfer map is natural. A consideration of the resulting commutative diagram completes the proof of the theorem.

Corollary 4.2. *Assume that the groups X' and X in Theorem* 4.3 *are the same. Then the Tate cohomology in the exact sequence* (4.4) *is isomorphic to $H^*(Y'/Y)$, where the involution is induced by $\gamma\alpha$.*

Proof. This follows from the commutative diagram in Theorem 4.3.

It is worth noting that the properties of the groups LP here are considerably different from the properties of the groups LS. It suffices to compare Corollary 4.1 and Example 1 in [11].

Corollary 4.3. *Assume that the groups Y' and Y in Theorem* 4.3 *are the same. Then the Tate cohomology in the exact sequence* (4.4) *is isomorphic to $H^{*+1}(X'/X)$, where the involution is induced by α.*

We recall that the Tate cohomology can be also realized on the spectral level (see [19]). In this case the Rothenberg exact sequence is the homotopy long exact sequence of the fibration (cofibration) of spectra. Let $\mathbb{H}(*)$ be the spectra with homotopy groups isomorphic to the Tate cohomology $H^n(*)$ (in a similar way to the $\mathbb{L}$-spectra; see also [11]).

We now consider the general case of a geometric diagram of antistructures Ψ depicted in (2.5) with two possible decorations k and l, $k \subset l$, of the following form:

$$\begin{array}{ccccccc} X & \longrightarrow & Z & & X' & \longrightarrow & Z' \\ \downarrow & & \downarrow & \subset & \downarrow & & \downarrow \\ Y & \longrightarrow & W & & Y' & \longrightarrow & W'. \end{array}$$

Theorem 4.4. *Under the above hypotheses we have the long exact sequence*

$$\longrightarrow LP^k_*(\Psi) \longrightarrow LP^l_*(\Psi) \longrightarrow \pi_*\big(\mathbb{H}((l/k)^!)\big) \longrightarrow,$$

in which the spectrum $\mathbb{H}((l/k)^!)$ is homotopy equivalent to a cofibre of any of the following three maps of spectra:

$$\Omega\mathbb{H}(f_*) \to \mathbb{H}(i^!_-), \quad \mathbb{H}(Y'/Y) \longrightarrow \mathbb{H}(Z'/Z), \quad \Omega\mathbb{L}(j^!_-) \to \mathbb{H}\big((g_\gamma)_*\big).$$

Proof. There is a natural map of the homotopy commutative diagram of spectra (3.1) equipped with decorations k into the same diagram with decorations l. We consider the cofibres of this map to obtain the homotopy commutative diagram

$$\begin{array}{ccc} \mathbb{H}(X'/X) & \longrightarrow & \mathbb{H}(Z'/Z) \\ \uparrow & & \uparrow \\ \mathbb{H}(Y'/Y) & \longrightarrow & \mathbb{H}(W'/W). \end{array}$$

We now extend this diagram in the vertical and the horizontal directions by cofibrations (see [11]) and define the spectrum $\mathbb{H}((l/k)^!)$ in the same way as we have defined the spectrum $\mathbb{LP}(\Psi)$. Our further arguments repeat the proof of Theorem 3.2 (see also [11]).

We note in conclusion that, as Theorem 4.4 shows, the groups $LP_*(\Psi)$ corresponding to the same geometric diagram of antistructures equipped with different decorations vary only in 2-torsion.

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