Obstructions to reconstructions from a pair of manifolds

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The groups LP_* of obstructions to reconstructions from a pair of manifolds $X \subset Y$ have been introduced geometrically by Wall (see [1]). Then an algebraic definition of the groups LP_* has been given by Ranicki as for the groups LS_* of obstructions to decomposition (see [2]). The groups LS_n and LP_n can be defined functorially by the universally repelling square F of the oriented fundamental groups and by $n \mod 4$. A pair (Y, X) is a Browder–Livesay pair if X is a one-sided submanifold of codimension 1 and the maps $\pi_1(\partial U) \to \pi_1(Y \setminus X)$, $\pi_1(X) \to \pi_1(Y)$ are isomorphisms, where U is a tube neighbourhood of X in Y. Then we have the isomorphisms

$$LS_n(F) \cong LN_n\big(\pi_1(Y \setminus X) \to \pi_1(Y)\big), \qquad LP_n(F) \cong L_{n+1}(i^!),$$

where $i^!\colon L_n\big(\pi_1(X) \to L_n\big(\pi_1(\partial U)\big)$ is a transfer.

If in this case the condition of isomorphism is weakened to epimorphism, then F is called a geometric diagram (see [3]). Then according to the above there is a close connection between the groups $LS_*(F)$ and the L-groups of the groups appearing in the diagram (see [4], [3]). In this case similar connections exist for the groups LP_*, as well.

For a geometric diagram F we denote by $j^!_-$ the transfer

$$L_n\big(\pi_1(Y)^-\big) \to L_n\big(\pi_1(Y \setminus X)\big),$$

where the sign '$-$' means that the group is considered with an altered orientation outside the image of j. We denote by δ_n the composition

$$L_{n+1}\big(\pi_1(\partial U) \to \pi_1(Y \setminus X)\big) \to L_n\big(\pi_1(\partial U)\big) \to L_n(i^!)$$

of the map from the relative exact sequence for the embedding $\pi_1(\partial U) \to \pi_1(Y \setminus X)$ and the map from the relative exact sequence for $i^!$. Similarly, we define the map δ'_n as the composition

$$L_{n+1}(j^!_-) \to L_n\big(\pi_1(Y)\big) \to L_n\big(\pi_1(X) \to \pi_1(Y)\big).$$

Theorem 1. *If F is a geometric diagram of groups, then the following long exact sequences hold:*

$$\longrightarrow L_{n+1}\big(\pi_1(\partial U)\big) \longrightarrow \pi_1(Y \setminus X)\big) \xrightarrow{\delta_n} L_n(i^!) \longrightarrow LP_{n-1}(F) \longrightarrow$$
$$\longrightarrow L_{n+1}(j^!_-) \xrightarrow{\delta'_n} L_n\big(\pi_1(X) \to \pi_1(Y)\big) \longrightarrow LP_{n-1}(F) \longrightarrow$$

The proof follows from a commutative diagram of spectra, which is extended to cofoliations:

$$F^! = \begin{array}{ccc} \boldsymbol{L}(\pi_1(\partial U)) & \longrightarrow & \boldsymbol{L}(\pi_1(Y \setminus X)) \\ \uparrow{\scriptstyle i^!} & & \uparrow{\scriptstyle j^!_-} \\ \boldsymbol{L}(\pi_1(X)) & \longrightarrow & \boldsymbol{L}(\pi_1(Y)) \end{array}$$

remaining homotopically commutative. About the $\boldsymbol{L}$-spectra we assume that $\pi_n(\boldsymbol{L}(*)) = L_n(*)$ and $\Omega \boldsymbol{L}_{n+1} = \boldsymbol{L}_n$, where the $\boldsymbol{L}_n$ are simplicial sets yielding the spectra.

As in [4], the diagram $F^!$ enables us to define the spectra $\boldsymbol{L}(F^!)$ and $\boldsymbol{LP}(F)$. Moreover, $\pi_n\big(\boldsymbol{LP}(F)\big) = LP_n(F)$.

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Theorem 2. *We have the following universal squares of spectra:*

$$\begin{array}{ccc} \Omega\boldsymbol{L}(\pi_1(\partial U)) & \longrightarrow & \Omega\boldsymbol{L}(\pi_1(Y\setminus X)) \\ \downarrow & & \downarrow \\ \Omega\boldsymbol{L}(i^!) & \longrightarrow & \boldsymbol{L}P(F) \end{array}$$

$$\begin{array}{ccc} \Omega\boldsymbol{L}(\pi_1(Y)^-) & \longrightarrow & \Omega\boldsymbol{L}(\pi_1(Y\setminus X)) \\ \downarrow & & \downarrow \\ \Omega\boldsymbol{L}(\pi_1(X)\to\pi_1(Y)^-) & \longrightarrow & \boldsymbol{L}P(F) \end{array}$$

$$\begin{array}{ccc} \Omega^2\boldsymbol{L}(F^!) & \longrightarrow & \Omega\boldsymbol{L}(\pi_1(X)\to\pi_1(Y)^-) \\ \downarrow & & \downarrow \\ \Omega\boldsymbol{L}(i^!) & \longrightarrow & \boldsymbol{L}P(F) \end{array}$$

The homotopic long exact sequences of the maps of the universal squares provide three Levin braids connecting the groups $LP_*(F)$ with the L-groups.

Bibliography

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