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On exact Milyutin mappings

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Abstract

We introduce the notion of an exact Milyutin mapping as a Milyutin mapping for which supports of values of its associated map coincide with point-preimages. We prove that every open continuous surjection between Polish spaces is an exact Milyutin mapping. For regular mappings we prove that the measure of singletons in the preimages equals zero for some exact Milyutin mapping. As a corollary, we obtain a proof of the local triviality of regular mappings with one-dimensional (not necessary compact) polyhedral fibers and a new proof of the same result for compact fibers.

Keywords: Milyutin mapping; Michael selection theorem; Paracompactness; Space of probabilistic measures; Polish space; Nonatomic measure

AMS classification: Primary 54C60; 54C65; 55R10, Secondary 54C10; 60B05; 28C15

1. Introduction

A continuous surjection $f: X \to Y$ between completely regular spaces X and Y is said to be a *Milyutin mapping* [12,15] if there exists a continuous mapping $\nu: Y \to P(X)$ such that for every point $y \in Y$,

$$\operatorname{supp} \nu_y \subset f^{-1}(y) \qquad (1.1)$$

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where $P(X)$ is the space of all probability measures with compact supports, endowed with the usual weak topology (see [9]), which is induced by the natural embedding of $P(X)$ into the Cartesian power $\mathbb{R}^{C(X)}$, where $C(X)$ is the space of all bounded continuous real-valued functions on X. Here, the *support* $\operatorname{supp}\mu$ of the measure μ is defined as the intersection of all closed subsets $F \subset X$ such that $\mu(B) = 0$, for every Borel set $B \subset X \setminus F$. Mappings of such type are also regarded from a more general descriptive set theory point of view. Namely, a map $\nu : Y \to P(X)$ is called a *transition kernel* provided that for every Borel set $B \subset X$ the map $y \mapsto \nu_y(B)$ is Borel measurable. A map $f : X \to Y$ is said to be *perfect statistic* for a transition kernel $\nu : Y \to P(Y)$, provided $\nu_y(f^{-1}(y)) = 1$ for each $y \in Y$, see [4,5].

In our earlier paper [12] we proved the following result:

Theorem 1.1. *Every paracompact space X is the image of some paracompact space X_0 of Lebesgue covering dimension $\dim X_0 = 0$, under a perfect Milyutin mapping $p : X_0 \to X$.*

In the present paper we prove that for every continuous open surjection $f : X \to Y$ between Polish spaces X and Y one can choose the map $\nu : Y \to P(X)$ so that the inclusion in condition (1.1) can be replaced by the equality:

$$\operatorname{supp}\nu_y = f^{-1}(y). \tag{1.2}$$

We shall call such f an *exact Milyutin* mapping. In the probability theory and statistics an analog of exact Milyutin mapping is a well-known notion, *a full perfect statistic*. As usual, a Polish space is a synonym for a separable completely metrizable space. Note, that Theorem 1.1 remains valid if "paracompact" is replaced by "Polish" (see [1]). Our main result states that each open continuous surjection f between Polish spaces is a full perfect statistic for a suitable continuous transition kernel ν.

Theorem 1.2. *Every continuous open surjection $f : X \to Y$ between Polish spaces X and Y is an exact Milyutin mapping.*

Since the proof of Theorem 1.2 uses, in an essential way, the Michael selection theorem, our approach does not allow a straightforward generalization beyond the class of completely metrizable spaces. The separability restriction is essential because of our use of the existence of a probability measure whose support coincides with the whole space.

Also, we shall prove that sometimes it is possible to unify condition (1.2) with the following condition:

$$\nu_y(\{x\}) = 0, \quad \text{for all } x \in f^{-1}(y). \tag{1.3}$$

We shall call such f an *atomless exact Milyutin* mapping.

Theorem 1.3. *Every topologically regular mapping $f : X \to Y$ between Polish spaces X and Y whose point-preimages are homeomorphic to a fixed Polish space without isolated points is an atomless exact Milyutin mapping.*

We say that continuous surjection $f : X \to Y$ *is topologically regular* (*completely regular* in [2]) if for every $y \in Y$ and $\varepsilon > 0$, there is a $\delta > 0$ such that if $d(y, y') < \delta$ then there is an ε-homeomorphism from $f^{-1}(y)$ onto $f^{-1}(y')$, i.e., a homeomorphism which moves points for a distance less than ε.

Corollary 1.4. *For every Polish space K there exists a continuous map $\mu : \exp K \to P(K)$ such that $\operatorname{supp} \mu(F) = F$, for every subcompactum $F \subset K$.*

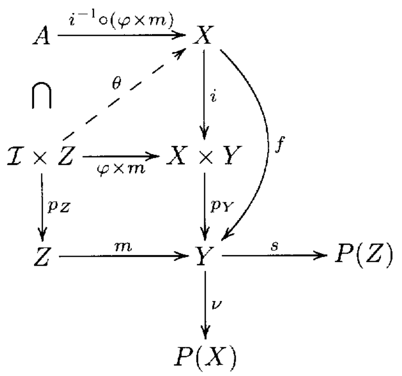
Corollary 1.5. *Every topologically regular mapping between Polish spaces whose preimages are homeomorphic to a fixed compact one-dimensional polyhedron is a locally trivial bundle.*

Corollary 1.6. *Every topologically regular mapping between Polish spaces whose preimages are homeomorphic to the real line is a locally trivial bundle.*

Corollary 1.5 generalizes the results of [11,13]: in [11] such a result was obtained for fibers homeomorphic to a finite graph such that the order of each vertex is different from two whereas in [13] such a result was proved for the compact case. We point out that in Corollaries 1.5 and 1.6 there are no dimensional restrictions for the range of the regular mapping. The authors wish to acknowledge the referee for several remarks and observations.

2. The construction of ν

We shall describe the construction of the map $\nu : Y \to P(X)$ which satisfies condition (1.2), i.e., such that $\operatorname{supp} \nu_y = f^{-1}(y)$. Consider the following main diagram:



(2.1)

Here:

(1) $\mathcal{I} = \mathbb{N}^\infty$ is the space of irrational numbers.

(2) $i : X \to X \times Y$ is an embedding which identifies X with the graph of the map f, i.e., $p_Y \circ i = f$, where $p_Y : X \times Y \to Y$ is the projection onto the second factor.

(3) m is a Milyutin mapping of a zero-dimensional metric space Z onto Y and the map s is *associated* to m, i.e., $\operatorname{supp} s_y \subset m^{-1}(y)$, $y \in Y$.
(4) For the construction of a pair of maps (m, s) in (3) one can use an embedding $j: Y \to Q$ of Y into Hilbert cube Q and the standard Milyutin map $m_0: C \to Q$ of the Cantor set C onto Q (see [8,10]). It then suffices to define $Z = m_0^{-1}(j(Y))$ and $m = m_0|_Z$. Note that all point-preimages $m^{-1}(y)$, $y \in Y$, are compact subsets of Z.
(5) φ is an arbitrary continuous surjection of $\mathcal{I}$ onto X and λ is a probability measure on $\mathcal{I}$ whose support $\operatorname{supp} \lambda$ coincides with $\mathcal{I}$ on $\mathbb{N}$ such a measure clearly exists while on $\mathcal{I} = \mathbb{N}^\infty$ one has to consider its countable power.
(6) $\varphi \times m: \mathcal{I} \times Z \to X \times Y$ is the Cartesian product of surjections φ and m and $A = (\varphi \times m)^{-1}(i(X))$, i.e., $A = \{(t, z) \mid \varphi(t) \in f^{-1}(m(z))\}$. Note that A is closed in $\mathcal{I} \times Z$ since $i(X)$ is closed in $X \times Y$ because of the openness of f.
(7) θ is a continuous selection of the lower semicontinuous multivalued map $\Theta: \mathcal{I} \times Z \to X$, given by

$$\Theta(t, z) = \begin{cases} \{\varphi(t)\}, & \text{if } (t, z) \in A, \\ f^{-1}(m(z)), & \text{if } (t, z) \notin A. \end{cases}$$

Such a selection exists by Michael's selection theorem [7], due to the 0-dimensionality of the space $\mathcal{I} \times Z$, completeness of values of Θ in X, closeness of A, openness of the map f and the fact that on A the map $(t, z) \mapsto \varphi(t)$ is a selection of the map given by $(t, z) \mapsto f^{-1}(m(z))$.
(8) $P(\theta): P(\mathcal{I} \times Z) \to P(X)$ is a map between the spaces of probability measures which is induced by the map $\theta: \mathcal{I} \times Z \to X$. Here, the value of the measure $[P(\theta)]\mu$ on the set $B \subset X$ is by definition equal to $\mu(\theta^{-1}(B))$, for every $\mu \in P(\mathcal{I} \times Z)$.
(9) $\nu_y = P(\theta)[\lambda \otimes s_y]$, where $\lambda \otimes s_y$ denotes the measure-product in $P(\mathcal{I} \times Z)$; $\lambda \in P(\mathcal{I})$ has $\operatorname{supp} \lambda = \mathcal{I}$ and $s_y \in P(Z)$, $y \in Y$, has $\operatorname{supp} s_y \subset m^{-1}(y) \subset Z$ (see (3) and (5)).

3. Proof of Theorem 1.2

By construction, we have that $[f \circ \theta](t, z) \in f(f^{-1}(m(z))) = m(z) = [m \circ p_Z](t, z)$, i.e., $f \circ \theta = m \circ p_Z$. Next, the continuity of the map $\nu: Y \to P(X)$ follows by the continuity of the maps $s|_Y$, $\lambda \otimes s_Y$, θ and the functoriality of P (see [3]).

Let's verify that for every $y \in Y$, $\operatorname{supp} \nu_y \subset f^{-1}(y)$. To this end we calculate the value of the measure ν_y on the set $B = f^{-1}(y) \subset X$. By definition, we have that

$$\begin{aligned} \nu_y(B) &= (P(\theta)[\lambda \otimes s_y])(B) = (\lambda \otimes s_y)(\theta^{-1}(f^{-1}(y))) \\ &= (\lambda \otimes s_y)(p_Z^{-1}(m^{-1}(y))) = (\lambda \otimes s_y)(\mathcal{I} \times m^{-1}(y)) \\ &= \lambda(\mathcal{I}) s_y(m^{-1}(y)) = 1, \end{aligned}$$

since $\operatorname{supp} s_y \subset m^{-1}(y)$, see (4). Therefore, the closed set $f^{-1}(y)$, has the property that for every $E \subset X \setminus f^{-1}(y)$, the value of the measure ν_y on E is equal to zero, i.e., $\operatorname{supp} \nu_y \subset f^{-1}(y)$.

Finally, let's prove that $\operatorname{supp} \nu_y = f^{-1}(y)$, for every $y \in Y$. This equality is equivalent to the property of the measure ν_y that its value on every nonempty open subset of preimage $f^{-1}(y)$ is positive. Let $G \subset X$ be an open subset of the space X intersecting the preimage $f^{-1}(y)$. Let's check that the set $\theta^{-1}(G \cap f^{-1}(y))$ has a subset of type $U \times m^{-1}(y)$, for some nonempty open set $U \subset \mathcal{I}$. We obtain that

$$\begin{aligned}\nu_y(G \cap f^{-1}(y)) &= (P(\theta)[\lambda \otimes s_y])(G \cap f^{-1}(y)) = [\lambda \otimes s_y](\theta^{-1}(G \cap f^{-1}(y))) \\ &\geqslant [\lambda \otimes s_y](U \times m^{-1}(y)) = \lambda(U) \cdot s_y(m^{-1}(y)) = \lambda(U) > 0,\end{aligned}$$

because $\operatorname{supp} \lambda = \mathcal{I}$, see (5).

By (7) the map $\theta : \mathcal{I} \times Z \to X$ makes a continuous choice via (t, z) from the sets $f^{-1}(m(z))$ and for pairs $(t, z) \in A$ and such a choice coincides with the point $\varphi(t)$. So, fix $y \in Y$ and pick any $x \in G \cap f^{-1}(y)$, $t \in \varphi^{-1}(x) \subset \mathcal{I}$. For every $z \in m^{-1}(y)$, we have that $(t, z) \in A$ and $\theta(t, z) = \varphi(t) = x$. By the continuity of selection θ at the point (t, z), we can find an open rectangle neighborhood $U \times V = (U \times V)(t, z)$ such that $\theta(U \times V) \subset G$. By compactness of the preimage $m^{-1}(y)$, we can find a finite cover of the set $\{t\} \times m^{-1}(y)$ by such open rectangles $\{U_i \times V_i\}_{i=1}^{n}$. Here, U_i are neighborhoods of the point $t \in \mathcal{I}$, $\{V_i\}_{i=1}^{n}$ is an open cover of the compactum $m^{-1}(y)$ and $\theta(U_i \times V_i) \subset G$. Let us now verify that

$$\left(\bigcap_{i=1}^{n} U_i\right) \times m^{-1}(y) \subset \theta^{-1}(G \cap f^{-1}(y)).$$

First, we have that

$$\theta\left(\left(\bigcap_{i=1}^{n} U_i\right) \times m^{-1}(y)\right) \subset \theta\left(\bigcup_{i=1}^{n} U_i \times V_i\right) \subset G.$$

Second, by the definition of the selection θ (see (7)), for every

$$(t', z) \in \left(\bigcup_{i=1}^{n} U_i\right) \times m^{-1}(y),$$

the value $\theta(t', z)$ lies in the set $f^{-1}(m(z)) = f^{-1}(y)$, i.e.,

$$\theta\left(\left(\bigcap_{i=1}^{n} U_i\right) \times m^{-1}(y)\right) \subset f^{-1}(y).$$

Thus we have checked the inclusion and this completes the proof of Theorem 1.2. □

Remark. Note, that there exists a direct way of calculating the value $\nu_y(B)$ of the measure ν_y over a Borel set $B \subset X$. To do this one must:

(a) for a fixed $z \in m^{-1}(y)$, find the preimage $(\theta|_{\mathcal{I} \times \{z\}})^{-1}(B) = B_z$;

(b) calculate the measure $\lambda(p_{\mathcal{I}}(B_z))$; and

(c) evaluate the integral

$$\int_{z\in m^{-1}(y)} \lambda\big(p_{\mathcal{I}}(B_z)\big)\,\mathrm{d}s_y.$$

4. Proof of Theorem 1.3

Step 1. We show that the case of an arbitrary Y can be reduced to the case $\dim Y = 0$. Let us consider the following diagram:

$$
\begin{array}{ccccccc}
X\times Z & \supset & T & \xrightarrow{p_X} & X & & \\
 & & \downarrow{\scriptstyle p_Z=p} & & \downarrow{\scriptstyle f} & & \\
 & & Z & \xrightarrow{m} & Y & \xrightarrow{s} & P(Z) \\
 & & \downarrow{\scriptstyle \lambda} & & \downarrow{\scriptstyle \nu} & & \\
 & & P(T) & & P(X) & &
\end{array} \tag{4.1}
$$

Here:

(1) The pair of maps (m, s) is as in (3) of Section 2.
(2) $T = \{(x, z) \in X \times Z\colon\ f(x) = m(z)\}$; p_X and $p_Z = p$ are projections onto the factors. Clearly, Z and T are Polish spaces.
(3) $\dim Z = 0$. Clearly the map p is an open surjection, and by hypothesis we can find a continuous map $\lambda\colon Z \to P(T)$ with properties (1.2) and (1.3), i.e.,

$$\operatorname{supp}\lambda_z = p^{-1}(z), \quad \text{for all } z \in Z, \quad \text{and}$$

$$\lambda_z\big(\{(x, z)\}\big) = 0, \quad \text{for all } (x, z) \in p^{-1}(z).$$

Now, for a fixed $y \in Y$, we consider a Borel set $B \subset f^{-1}(y)$ and for every $z \in m^{-1}(y)$, we consider the value $\lambda_z(B_z) \in [0, 1]$ of the measure λ_z on the Borel subset $B_z = \{(x, z)\colon\ x \in B\}$ of the preimage $p^{-1}(z)$. Then we put

$$\nu_y(B) = \int_{z\in m^{-1}(y)} \lambda_z(B_z)\,\mathrm{d}s_y.$$

If G is open in $f^{-1}(y)$, then G_z is open in $p^{-1}(z)$, for any $z \in m^{-1}(y)$ and hence $\lambda_z(G_z) > 0$. By the properties of the integral it follows that $\nu_y(G) > 0$.

If B is a singleton in $f^{-1}(y)$, the B_z is a singleton in $p^{-1}(z)$ and hence $\lambda_z(B_z) = 0$. So, $\nu_y(B) = 0$. This completes the proof of Step 1. Note, that we have used only openness of f, but not also the regularity of f.

Step 2. Let us prove Theorem 1.3 for zero-dimensional Polish spaces Y. Let $C(\mathcal{I}\times Y, X)$ be the set of all continuous mappings from $\mathcal{I} \times Y$ into X, endowed with the topology of uniform convergence. Then $C(\mathcal{I} \times Y, X)$ is a completely metrizable space. Let

$$S = \big\{s \in C(\mathcal{I} \times Y, X)\colon\ s\big(\mathcal{I} \times \{y\}\big) = f^{-1}(y) \text{ for all } y \in Y\big\}.$$

As in the proof of Theorem 1.2 we can see that S is nonempty. Clearly, the space S of all "fiberwise" mappings of $\mathcal{I}\times Y$ onto X is closed in $C(\mathcal{I}\times Y, X)$. Hence S is completely metrizable space, too. For each $s\in S$ and each $y\in Y$, let ν_y^s be the probability measure on the fiber $f^{-1}(y)$, defined as follows:

$$\nu_y^s(B) = \lambda\big(p_{\mathcal{I}}\big[(s|_{\mathcal{I}\times\{y\}})^{-1}(B)\big]\big),$$

where B is a Borel subset of X and $\lambda\in P(\mathcal{I})$, with $\operatorname{supp}\lambda = \mathcal{I}$. Clearly, $\operatorname{supp}(\nu_y^s) = f^{-1}(y)$, for all $y\in Y$, because $s\in S$.

Now, we define a multivalued mapping $H:Y\to S$ as follows: for each $y\in Y$, let $H(y)$ be the set of all mappings $s\in S$ such that the probability measure ν_y^s is atomless, i.e., $\nu_y^s(\{x\}) = 0$, for all $x\in f^{-1}(y)$. Clearly, $H(y)$ is a closed subset of S. For the mapping H, Michael's zero-dimensional selection theorem applies. Lower-semicontinuity of H follows by standard methods [2] from the regularity of f. Some technical difficulties arise, however, with the nonemptiness of $H(y)$, $y\in Y$. First, we represent the preimage $f^{-1}(y)$ as an image $\mathcal{I}\times\{y\}$ under some surjection which induces an atomless measure on $f^{-1}(y)$. Then we extend such a surjection to some element $s\in S$ in the same manner as we constructed the map θ in Section 2 above.

So, let $h:Y\to S$ be a continuous singlevalued selection of H, $h_y\in H(y)$. Then the map $m:\mathcal{I}\times Y\to X$, defined by

$$m(t,y) = h_y(t,y)$$

gives the desired atomless exact mapping $\nu:Y\to P(X)$, according to the formula above, i.e.,

$$\nu_y(B) = \lambda\big(p_{\mathcal{I}}\big[(m|_{\mathcal{I}\times\{y\}})^{-1}(B)\big]\big),\quad B\subset f^{-1}(y).$$

Indeed, $h_y\in S$ and hence

$$h_y(\mathcal{I}\times\{y\}) = f^{-1}(y),$$

i.e., $\operatorname{supp}(\nu_y) = f^{-1}(B)$ and from $h_y\in H(y)$ we conclude that ν_y is atomless. □

5. Proofs of corollaries

Proof of Corollary 1.4. Recall that $\exp K$ is the family of all nonempty subcompacta of the Polish space K, equipped with the Hausdorff distance topology with respect to which $\exp K$ is a Polish space, too (see [6, Theorem (7.5)]). Apply Theorem 1.2 for the spaces $Y = \exp K$,

$$X = \{(t,F):\ F\in\exp K,\ t\in F\}\subset K\times\exp K$$

and for the map $f:X\to Y$ be the restriction of the projection $p:K\times\exp K\to\exp K$ onto the second factor. Then for every $F\in Y=\exp K$, we obtain a probability measure $\mu(K\times\exp K)$, continuously depending on F, whose support coincides with the set $f^{-1}(F)$. Clearly, under the projection of X onto the first factor of the product $K\times\exp K$, the set $f^{-1}(X)$ is mapped homeomorphically precisely onto the set F. Therefore, we have constructed the desired mapping of $\exp K$ into $P(K)$. □

Proof of Corollary 1.5. For simplicity let us consider the case of the unit interval as the fiber. Let $y_0 \in Y$, let $\{c_0, d_0\}$ be the endpoints of the preimage $f^{-1}(y_0)$ and let $2\varepsilon_0 = \mathrm{dist}(c_0, d_0) > 0$. Find a δ-neighborhood $U = U(y_0)$ such that for every $y \in U$, the preimages $f^{-1}(y_0)$ and $f^{-1}(y)$ are homeomorphic under some ε_0-homeomorphism. Then we can distinguish the endpoints of the preimages $f^{-1}(y)$, $y \in U$. One of these endpoints lies near c_0 and the other one lies near d_0. We denote these endpoints by $c(y)$ and $d(y)$, respectively.

By Theorem 1.3, there exists a continuous map $\nu: Y \to P(X)$ such that

$$\operatorname{supp} \nu_y = f^{-1}(y), \quad y \in Y, \tag{5.1}$$

and

$$\nu_y(\{x\}) = 0, \quad x \in f^{-1}(y). \tag{5.2}$$

Now, for every $x \in f^{-1}(U)$ we put

$$\psi(x) = \big(f(x), \nu_{f(x)}\big([c(f(x)), x]\big)\big) \in U \times [0,1]$$

where we denoted with $[c(f(x)), x]$ the part of the arc $f^{-1}(f(x))$ between the points $c(f(x))$ and x. In order to prove the bijectivity of the map $\psi: f^{-1}(U) \to U \times [0,1]$ it is sufficient to observe that for a fixed $y \in U$ the map $\varphi_y(x) = \nu_y([c(y), x])$, $\varphi_y: f^{-1}(y) \to [0,1]$, is monotone because the measure ν_y is a monotone function of sets. From (5.1) we obtain that φ_y is strongly monotone, i.e., if $[c(y), x] \subset [c(y), x']$, $x \neq x'$, then $\varphi_y(x) < \varphi(x')$. From (5.2) we conclude that φ_y is in fact a continuous function and hence φ_y is a bijection. Continuity of the map follows from the continuity of f, ν and $c|_U$. □

For an arbitrary, compact one-dimensional polyhedron an argument, similar to the one in [14], can be used.

Proof of Corollary 1.6. We repeat the idea of the previous proof. However, we start from the points $c(y)$, $y \in U(y_0)$, which divide the point-preimages $f^{-1}(y)$ into two "equal" parts. This means that $f^{-1}(y) \backslash c(y)$ has exactly two connected components and the values of measures ν_y at this components are equal to $1/2$.

The existence of such an intermediate point $c(y)$ follows from condition (5.2) of the atomlessness of measures ν_y and the uniqueness of such points follows from condition (5.1) of exactness of measures ν_y. □

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