Graduate Workshop in Mathematics
and Its Applications,
Ljubljana, 23.–27. 9. 1991

A CRITERION FOR THE ENDPOINT COMPACTIFICATION OF AN OPEN 3-MANIFOLD WITH ONE END TO BE A GENERALIZED 3-MANIFOLD

D. REPOVŠ

Math. Subj. Class. (1991) Primary: 57N10, 57N13, 54D35
Secondary: 57P05, 57N99, 54D05

Key words and phrases: open 3-manifold, ends, endpoint compactification, generalized 3-manifold, pushing loops close to infinity, local simple connectivity

Abstract: We study the following problem: to find conditions, "checkable from within" an open 3-manifold M, which guarantee that the endpoint compactification $\hat{M}$ of M is a generalized 3-manifold. Our main result is: The endpoint compactification $\hat{M}$ of a 3-manifold M with one end is a generalized 3-manifold if and only if M satisfies the property that (1) given a neighbourhood U of ∞ there exists a neighbourhood $V \subset U$ of ∞ such that for every $k = 2, 3$ and for every mapping $f : \partial B^k \to V$ and every neighbourhood $W \subset V$ of ∞, there exist pairwise disjoint k-cells $D_1, ..., D_t \subset \operatorname{int} B^k$ and a mapping $F : D \to U$ such that $D = \overline{B^k - (D_1 \cup ... \cup D_t)}$, $F \mid \partial B^k = f$, and $F(\partial D_j) \subset W$ for every $j \in \{1, ..., t\}$; (2) $H_2(\_; \mathbf{Z})$ is stable at the end ϵ; and (3) $H_2(\epsilon; \mathbf{Z}) \cong \mathbf{Z}$.

Assume throughout this paper that M is a topological 3-manifold with the following properties: (i) M is noncompact; (ii) ∂M is either compact or empty; (iii) M has one end; and (iv) M contains no fake 3-cells.

Recall the definition of an *end* of a locally compact space X: this is a collection E of open subsets of X satisfying the following properties: (1) Each element of E is open, connected, and nonempty; (2) Each element of E has compact frontier; (3) If $e_1, e_2 \in E$ then there is $e_3 \in E$ such that $e_3 \subset e_1 \cap e_2$; (4) $\cap\{\bar{e} \mid e \in E\} = \emptyset$; and (5) E is maximal with respect to properties (1)–(4).

A prime example is $W = N - C$, where N is a compact topological manifold with boundary and $C \subset \partial N$ is a boundary component. Then W has exactly one end [8] [9] [14].

Denote by $\hat{M}$ the endpoint (Freudenthal) compactification of M and let $\hat{M} - M = \{\infty\}$. The following theorem was first proved by C. H. Edwards, Jr. [7] and, independently, by C. T. C. Wall [16]:

Theorem 1. (C. H. Edwards and C. T. C. Wall) *$\hat{M}$ is a 3-manifold if and only if M is simply connected at ∞.*

A *neighbourhood of infinity* in a locally compact space X is an open set $U \subset X$ such that $X - U$ is compact. A locally compact space X is *simply connected at* ∞ if for every neighbourhood $U \subset X$ of ∞ there exists a neighbourhood $V \subset U$ of ∞ such that every loop in V is null-homotopic in U. Thus M is simply connected at ∞ if and only if $\{\infty\}$ is 1-LCC in $\hat{M}$.

Note the distinction between these properties and assertion that $\hat{M}$ is 1-LC at ∞: $\{\infty\}$ is 1-LCC in $\hat{M}$ if for every open set $\hat{U}$ in $\hat{M}$ there exists an open set $\hat{V} \subset \hat{U}$ such that the inclusion-induced homomorphism $\Pi_1(\hat{V} - - \{\infty\}) \to \Pi_1(\hat{U} - \{\infty\})$ is zero. On the other hand, $\hat{M}$ is 1-LC at ∞ if and only if for every open set $\hat{U} \subset \hat{M}$ there is an open set $\hat{V} \subset \hat{U}$ such that $\Pi_1(\hat{V}) \to \Pi_1(\hat{U})$ is zero.

The question which we wish to address here is: Are there conditions "checkable from within M" that are collectively equivalent to $\hat{M}$ being a *generalized 3-manifold*, i.e. a locally compact, finite-dimensional, separable metrizable ANR which is also a $\mathbf{Z}$-homology 3-manifold (i.e. for every $x \in \hat{M}$, $H_*(\hat{M}, \hat{M} - \{x\}; \mathbf{Z}) \cong H_*(\mathbf{R}^3, \mathbf{R}^3 - \{0\}; \mathbf{Z})$)?

One approach to this problem is to break the statement "$\hat{M}$ is a generalized 3-manifold" into simpler properties and search for solutions to the problem using these more basic properties. For example, $\hat{M}$ is clearly finite-dimensional, so $\hat{M}$ is an ANR if and only if $\hat{M}$ is locally contractible at ∞ [2]. Now, $\hat{M}$ is clearly 0-locally connected (0-LC) at ∞. Since $\hat{M}$ deforms to the one-point compactification of a locally finite 2-dimensional polyhedron (an unpublished result of G. Kozlowski), LC^2 implies LC^∞ [11]. Therefore, $\hat{M}$ is an ANR if and only if $\hat{M}$ is 1-LC and 2-LC at ∞. (Recall that X is k-LC at $x \in X$ if for every neighbourhood $U \subset X$ of x there is a neighbourhood $V \subset U$ of x such that $\Pi_k(V) \to \Pi_k(U)$ is zero, and LC^k means n-LC for all $n \leq k$.)

Furthermore, using the local version of the Hurewicz theorem [11], the property 2-LC may be substituted by its homological equivalent, 2-lc, if it is desirable. Similarly, it can be shown that $\hat{M}$ is a $\mathbf{Z}$-homology 3-manifold if and only if $H_q(\hat{M}, M; \mathbf{Z}) \cong H_q(\mathbf{R}^3, \mathbf{R}^3 - \{0\}; \mathbf{Z})$ for $1 \leq q \leq 3$. Can each of these more basic conditions be recognized from within M?

First, we consider such a criterion for the local k-connectedness of $\hat{M}$, due to J. Dydak [5] (see also [6]). It will be called the $\mathrm{PS}^k\mathrm{CI}$ *property* (for "Pushing k-Spheres Close to Infinity"): M has the $\mathrm{PS}^k\mathrm{CI}$ *property* if, given a neighbourhood $U \subset M$ of ∞ there exists a neighbourhood $V \subset U$ of ∞ such that for every mapping $f : \partial B^{k+1} \to V$ and every neighbourhood $W \subset V$ of ∞ there exist pairwise disjoint $(k+1)$-cells $D_1, ..., D_t \subset \operatorname{int} B^{k+1}$ and a mapping $F : D \to U$ such that $D = \overline{B^{k+1} - (D_1 \cup ... \cup D_t)}$, $F \mid \partial B^{k+1} = = f$, and $F(\partial D_j) \subset W$ for every $j = 1, ..., t$. For example, M has the $\mathrm{PS}^k\mathrm{CI}$ property if and only if for every neighbourhood U of infinity there exists a neighbourhood of infinity $V \subset U$ such that for every neighbourhood $W \subset V$

of infinity, loops in V are freely homotopic within U to a product of loops in W.

Theorem 2. *Let M be a noncompact 3-manifold with ∂M either empty or compact with one end and let $k \in \{1,2\}$. Then the endpoint compactification $\hat{M}$ of M is k-LC at ∞ if and only if M has the property* $\mathrm{PS}^k\mathrm{CI}$.

Proof of Theorem 2. Theorem 2 follows immediately by [5; Lemma (3.2)]. Nevertheless, for the sake of exposition we present here a detailed proof of the $k = 1$ case. Suppose first that $\hat{M}$ is 1-LC at ∞. Given a neighbourhood U of infinity let $\hat{U} = U \cup \{\infty\}$. Since $\hat{M}$ is 1-LC at ∞, there exists a neighbourhood $\hat{V}$ of ∞ in $\hat{M}$ such that any loop in $\hat{V}$ is null-homotopic in $\hat{U}$. Let $V = \hat{V} - \{\infty\}$.

Let $f : \partial B^2 \to V$ be a mapping and W a neighbourhood of ∞. Let $F : B^2 \to \hat{U}$ be an extension of f. Choose a polyhedral manifold neighbourhood N of $F^{-1}(\infty)$ in B^2, small enough so that $N \subset F^{-1}(W)$. Let D be the component of $\overline{B^2 - N}$ containing ∂B^2 and define $G = F \mid D$. D is a disk-with-holes as in the definition of $\mathrm{PS}^1\mathrm{CI}$ and $G(\partial D - \partial B^2) \subset W$. Therefore M has the $\mathrm{PS}^1\mathrm{CI}$ property.

Suppose now that M has the $\mathrm{PS}^1\mathrm{CI}$ property and let U be a neighbourhood of ∞ in $\hat{M}$. Let $U = \hat{U} - \{\infty\}$, and let $V \subset U$ be a neighbourhood of ∞ as in the definition of $\mathrm{PS}^1\mathrm{CI}$. Finally, let $\hat{V} = V \cup \{\infty\}$. Clearly, $\hat{V}$ is a neighbourhood of ∞ in $\hat{M}$, and it remains to be shown that any mapping $f : \partial B^2 \to V$ can be extended to a mapping $F : B^2 \to U$.

As a special case, assume $f(\partial B^2) \subset V$. Let $U_0 = U$, $U_1 = V$, and in general, let U_{n+1} be a neighbourhood of ∞ such that the pair (U_{n+1}, U_n) satisfies the requirements for (U, V) in the definition of $\mathrm{PS}^1\mathrm{CI}$. Furthermore, construct the U_j's so that $\{\hat{U}_j\}_{j\in\mathrm{N}}$ is a neighbourhood basis for M at ∞. Extend f to a mapping $f_1 : D_1 \to U$, where D_1 is a disk-with-holes in B^2 and $f_1(\partial D_1 - \partial B^2) \subset U_2$. Inductively, extend f_n to a mapping $f_{n+1} : D_{n+1} \to U_1$, where D_{n+1} is a disk-with-holes in B^2, $(D_n - \partial B^2) \subset \operatorname{int} D_{n+1}$, $f_{n+1}(D_{n+1} - D_n) \subset U_{n-1}$ and $f_{n+1}(\partial D_{n+1} - \partial B^2) \subset U_{n+1}$. The disk-with-holes D_{n+1} should be constructed so that the components of $B^2 - D_{n+1}$ have diameters $< \frac{1}{n+1}$, so that $D_\infty = \bigcup_{n\geq 1} D_n$ is the complement of a 0-dimensional compactum in $\operatorname{int} B^2$. Define $f_\infty : D_\infty \to U$ by $f_\infty \mid D_n = f_n$, $n \in \mathbf{N}$. Then f_∞ is a proper mapping of D_∞ into U, with the ends of D_∞ all going to the end of U at ∞. Therefore $F \mid D_\infty = f$ and $F(B^2 - D_\infty) = \infty$, defines a mapping of B^2 into $\hat{U}$ that extends f.

Now let $f : \partial B^2 \to \hat{V}$ be an arbitrary mapping $K = f^{-1}(\infty)$. Let $V_1 \supset V_2 \supset \ldots$ be connected neighbourhoods of ∞, chosen so that $\{\hat{V}_j\}_{j\in\mathrm{N}}$ is a basis for $\hat{M}$ at ∞ and so that any loop in V_{j+1} is null-homotopic in $\hat{V}_j$, as in the Special case. Recall that any loop in V is null-homotopic in $\hat{U}$, so we may set $V_1 = V$ and $V_0 = U$.

The complement of K in B^2 may be written as the union of 2-cells $B_1 \subset B^2 \subset \ldots$, where $(B_j \cap \partial B^2) \cup f^{-1}(V_j) = \partial B^2$. Using connectivity of V_j, f may be extended over $\partial B_j - \partial B^2$ so that $f(\partial B_j - \partial B^2) \subset V_j$. Let $C_j = B_{j+1} - B_j$ and $C_0 = B_1$. Then C_j is a union of 2-cells and $f(\partial C_j) \subset V_j$ for each j. Applying the Special case to $f \mid \partial C_j$, we extend f to a mapping of C_j into $\hat{V}_{j-1}$ for each j, resulting in an extension of f to a mapping $F : B^2 \to \hat{V}_0$. ∎

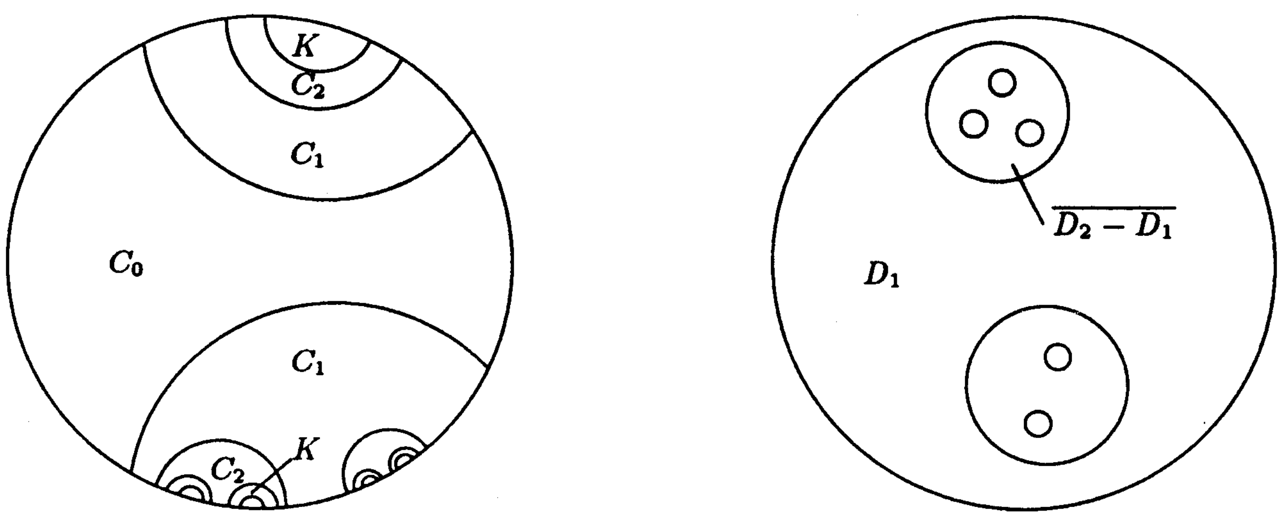

Figure 1

Before we continue we need to introduce a new concept — stability of homology groups at an end, and following that, homology groups of an end. So let X be a locally compact space with one end ε. Following [14], we shall define $H_*(\varepsilon; \mathbf{Z})$ to be the inverse limit of the inverse system $\{H_*(U_i; \mathbf{Z}); \alpha_{i,i+1}\}_{i \in \mathrm{N}}$, associated to a system of open neighbourhoods

$$U_1 \xleftarrow{\alpha_{1,2}} U_2 \xleftarrow{\alpha_{2,3}} U_3 \xleftarrow{\alpha_{3,4}} \ldots$$

of the end ε, which is *stable*, i.e. for some subsequence $\{H_*(U_{i_j}; \mathbf{Z}); \alpha_{i_j, i_j+1}\}_{j \in \mathrm{N}}$, the induced maps are isomorphisms:

$$\cdots \longleftarrow \operatorname{im}\alpha_{i_1,i_2} \xleftarrow[\cong]{\alpha_{i_1,i_2}|} \operatorname{im}\alpha_{i_2,i_3} \xleftarrow[\cong]{\alpha_{i_2,i_3}|} \cdots$$

It can be shown, using the same ideas as in [14] that $H_*(\varepsilon; \mathbf{Z})$ is well-defined.

We define two more properties. Let X be any space. Then X is said to satisfy the *Kneser finiteness* if no compact subset of X intersects more than a finite number of pairwise disjoint fake 3-cells. Next, X is said to have the *map separation property* if for every collection $f_1, \ldots, f_n : B^2 \to X$ of Dehn disks such that if $i \neq j$ then $f_i(B^2) \cap f_j(\operatorname{int} B^2) = \emptyset$ and for every neighbourhood $U \subset X$ of the set $\bigcup_{i=1}^n f_i(B^2)$ there exist maps $g_1, \ldots, g_n : B^2 \to U$ such that (i) for every i, $f_i \mid \partial B^2 = g_i \mid \partial B^2$; and (ii) for every $i \neq j$, $g_i(B^2) \cap g_j(B^2) = \emptyset$. Recall that a disk $f : B \to X$ is said to be *Dehn* if the closure of the set $\{x \in B^2 \mid f^{-1}(f(x)) \neq x\}$ misses ∂B^2. For more on these properties see [13].

We now come to the main result of the paper — an interior characterization of generalized 3-manifolds:

Theorem 3. *Let M be an open 3-manifold with one end ε. Then $\hat{M}$ is a generalized 3-manifold if and only if the following conditions hold:*

(i) *for every neighbourhood $U \subset M$ of ∞ there is a neighbourhood $V \subset U$ of ∞ such that for every map $f : \partial B^k \to V$, $k = 2, 3$, and every neighbourhood $W \subset M$ of ∞ there exist k-cells $B_1^k, \ldots, B_m^k \subset B^k$ and an extension*

$$f' : (B^k - \bigcup_{i=1}^{m} \operatorname{int} B_i^k) \to U$$

of f, such that $(\operatorname{int} B_i^k) \cap (\operatorname{int} B_j^k) = \emptyset$ for all $i \neq j$ and $f'(\partial B_i^k) \subset W$ for all $i \leq m$; and

(ii) *$H_2(\_; \mathbf{Z})$ is stable at ε and $H_2(\varepsilon; \mathbf{Z}) \cong \mathbf{Z}$.*

Proof. We only need to prove the sufficiency. Clearly, $\hat{M}$ is always finite-dimensional since such is already M, so $\hat{M}$ is an ENR if and only if $\hat{M}$ is LC^∞ at ∞ [2]. Now, $\hat{M}$ is always LC^0 at ∞ and since $\hat{M}$ deforms onto a one-point compactification of some locally finite 2-dimensional polyhedron (as observed by G. Kozlowski) it suffices to prove that $\hat{M}$ is LC^2 at ∞. The latter is by Theorem 2 precisely the condition (i) above.

Next, by the Hurewicz theorem, M is 1-lc $(\mathbf{Z})$ at ε. Let $\{\hat{U}_i\}$ be a neighbourhood base at ∞. Consider the long exact sequence for the Borel-Moore homology [1] with compact supports for the pair $(\hat{U}_i, \hat{U}_i - \{\infty\})$:

$$\cdots \to H_k^c(\hat{U}_i) \to H_k^c(\hat{U}_i, \hat{U}_i - \{\infty\}) \to H_{k-1}^c(\hat{U}_i - \{\infty\}) \to H_{k-1}^c(\hat{U}_i) \to \cdots$$

Then by the Sklyarenko theorem [15], $\overleftarrow{\lim_i}\, H_*^c(\hat{U}_i) \cong 0 \cong \overleftarrow{\lim_i}{}^1 H_k^c(\hat{U}_i)$. Now, by excision, $H_k^c(\hat{U}_i, \hat{U}_i - \{\infty\})$ doesn't depend on the choice of U_i. It therefore follows by the condition (ii) of the theorem that

$$H_3^c(\hat{M}, \hat{M} - \{\infty\}) \cong H_3^c(\hat{U}_i, \hat{U}_i - \{0\}) \cong \overleftarrow{\lim_i}\, H_2^c(\hat{U}_i - \{\infty\}) \cong H_2^c(\varepsilon) \cong \mathbf{Z}.$$

Similarly, for $k \leq 2$, $H_k^c(\hat{M}, \hat{M} - \{\infty\})$ belongs to the short exact sequence

$$0 \longrightarrow \overleftarrow{\lim_i}{}^1 \tilde{H}_{k-1}^c(\hat{U}_i - \{\infty\}) \longrightarrow H_k^c(\hat{M}, \hat{M} - \{\infty\}) \longrightarrow$$
$$\longrightarrow \overleftarrow{\lim_i}\, \tilde{H}_k^c(\hat{U}_i - \{\infty\}) \longrightarrow 0$$

and the condition (i) implies that $\{\tilde{H}_1^c(\hat{U}_i - \{\infty\})\}$ vanishes.

It follows that: $H_i(\hat{M}, \hat{M} - \{\omega\}; \mathbf{Z}) \cong H_i(\mathbf{R}^3, \mathbf{R}^3 - \{0\}; \mathbf{Z})$, $i = 1,2,3$. Thus $\hat{M}$ is also a $\mathbf{Z}$-homology 3-manifold hence a generalized 3-manifold. ∎

Remark. If the Poincaré conjecture is true then the one-point compactification of an open 3-manifold M with one end need not be a generalized 3-manifold *even* if M is contractible. Let M be Kister-McMillan's open 3-manifold [10]. Then M is contractible and has one end. Suppose $\hat{M}$ were a generalized 3-manifold. Then by M. G. Brin [3] $\hat{M}$ would have a resolution so by Brin-McMillan [4] M would embed in a compact 3-manifold. However, the latter is known to be false.

If we add a general position hypothesis to Theorem 3, we get the following recognition theorem for 3-manifolds, by invoking the main theorem of [12]:

Theorem 4. *Let M be an open 3-manifold with one end ε. Then $\hat{M}$ is a topological 3-manifold if and only if the following conditions are satisfied:*

(i) *$\hat{M}$ satisfies the Kneser finiteness;*

(II) *$\hat{M}$ possesses the map separation property;*

(iii) *M satisfies the $PS^k CI$ property for $k = 1$ and 2; and*

(iv) *$H_2(\_; \mathbf{Z})$ is stable at ε and $H_2(\varepsilon; \mathbf{Z}) \cong \mathbf{Z}$.* ∎

We shall conclude with the following open problem. Let X be a connected ENR with one end. Let $U, W \subset X$ be open neighbourhoods of infinity such that W is connected and $W \subset U$. Let $x_0 \in U$ and $x_1, x_2 \in W$. Then there are paths γ_1 from x_1 to x_0, γ_2 from x_1 to x_0, and γ_0 from x_1 to x_2, and γ_0 lies in W. The inclusions induce isomorphisms $(i_k)_\# : \Pi_1(W, x_k) \to \Pi_1(U, x_k)$ for $k = 1,2$. The maps γ_k induce isomorphisms $(\gamma_j)_\# : \Pi_1(U, x_j) \to \Pi_1(U, x_0)$, $j = 1,2$ and $(\gamma_0)_\# : \Pi_1(W, x_1) \to \Pi_1(W, x_2)$. Let $\phi = (\gamma_2)_\#(\gamma_0)_\#(\gamma_1)_\#^{-1} : \Pi_1(U, x_0) \to \Pi_1(U, x_0)$. Then ϕ is an inner automorphism. Let $H_k = ((\gamma_k)_\#^{-1}(i_k)_\#)\Pi_1(W, x_n)$, $k = 1,2$ and let N_k be the normal closure of H_k in $\Pi_1(U, x_0)$. Then $N_1 = N_2$. Therefore, if we let G_W be the normal closure of $\text{im}[\Pi_1(W, x_1) \to \Pi_1(U, x_0)]$, then G_W is well-defined and we may set $\Pi_1^\infty(U, x_0) = \bigcap\{G_W \mid W \text{ open, connected } U\}$. We define thad X has the property PS^1I (for "Pushing 1-Spheres to Infinity") if for every open neighbourhood $U \subset X$ of ∞ there is an open neighbourhood $V \subset U$ of ∞ such that $\text{im}[\Pi_1(V) \to \Pi_1(U)] \subset \Pi_1^\infty(U)$ where we restrict to those W in the definition of $\Pi_1^\infty(U)$ which lie in V. Clearly, the PS^1CI property implies the PS^1I property. Does the converse also hold?

Acknowledgements. The preliminary version of this paper was written in 1979 (at which time the author was unaware of the related manuscript of J. Dydak [5] which was published in 1981). Taking into account the papers [5] and [6], the final revision was then made during the visit to the University of Nantes, France in February 1992. The author would like to acknowledge the kind hospitality of Professor H. Ibisch.

References

[1] A. Borel and J. C. Moore, *Homology theory for locally compact spaces,* Michigan Math. J. **7** (1960), 137–159.

[2] K. Borsuk, *Theory of Retracts,* Monogr. Mat.44, PWN, Warsaw 1967.

[3] M. G. Brin, *Generalized 3-manifolds whose nonmanifold set has neighbourhoods bounded by tori,* Trans. Amer. Math. Soc. **264** (19881) 539–555.

[4] M. G. Brin and D. R. McMillan, Jr., *Generalized three-manifolds with zero-dimensional nonmanifold set,* Pacif. J. Math. **97** (1981) 29–58.

[5] J. Dydak, *Local n-connectivity of quotient spaces and one-point compactifications,* Proc. Shape Theory and Geom. Topol. Dubrovnik 1981, S. Mardešić and J. Segal, Eds., Lect. Notes Math. **870**, Springer-Verlag, Berlin 1981, 48–72.

[6] J. Dydak and J. Segal, *Local n-connectivity of decomposition spaces,* Topol. Appl. **18** (1984) 43–58.

[7] C. H. Edwards, Jr., *Open 3-manifolds which are simply connected at infinity,* Proc. Amer. Math. Soc. **14** (1963) 391–395.

[8] D. B. A. Epstein, *Ends,* Topology of 3-Manifolds and Related Topics, M. K. Fort, Ed., Prentice-Hall, Englewood Cliffs, N. J. 1962, 110–117.

[9] H. Freudenthal, *Über die Enden topologischer Räume und Gruppen,* Math. Zeitschrift **33** (1931) 692–713.

[10] J. M. Kister and D. R. McMillan, Jr., *Locally euclidean factors of E^4 which cannot be embedded in E^3,* Ann. of Math. (2) **76** (1962) 541–546.

[11] R. C. Lacher, *Cell-like mappings and their generalizations,* Bull. Amer. Math. Soc. **83** (1977) 495–553.

[12] D. Repovš, *The recognition problem for topological manifolds, Geometric and Algebraic Topology,* J. Krasinkiewicz, S. Spież and H. Toruńczyk, Banach Math. Centre Publ. **18** Eds., PWN, Warsaw 1986, 77–108.

[13] D. Repovš, *Isolated singularities in generalized 3-manifolds,* C. R. Acad. Bulg. Sci. **39** (1986) 10, 13–15 and **40** (1987)3, 143.

[14] L. C. Siebenmann, *The obstruction to finding a boundary for an open manifold of dimensions greater than five,* Ph. D. Thesis, Princeton University, Princeton 1965.

[15] E. G. Sklyarenko, *Homology theory and the exactness axiom* (in Russian), Uspekhi mat. nauk **24** (1969)5, 87–140 = Russian Math. Surveys **24** (1969)5,91–142.

[16] C. T. C. Wall, *Open 3-manifolds which are 1-connected at infinity,* Quart. J. Math. (Oxford) (2) **16** (1965) 263–268.

D. Repovš
Institute for Mathematics,
Physics and Mechanics,
University of Ljubljana,
Ljubljana, Slovenia