Differential Inclusions and Optimal Control
Lecture Notes in Nonlinear Analysis
Volume 2, 1998, pages 253–262

CONTINUOUS SELECTIONS OF NON-LOWER SEMICONTINUOUS NONCONVEX-VALUED MAPPINGS

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1. Introduction

While lower semicontinuity of mappings with closed convex values is ***sufficient*** for the existence of continuous singlevalued selections, it is of course, not *necessary*. For example, one can start by an arbitrary continuous singlevalued mapping $f : X \to Y$ and then define $F(x)$ to be a subset of Y such that $f(x) \in F(x)$. Then F admits the selection f, but there are no continuity type restrictions for F. A very natural problem immediately arises. Namely, to find a weaker version of lower semicontinuity which preserves the existence of singlevalued selections. If we can find a lower semicontinuous selection G of a given convex-valued mapping F, then Michael's techniques can be used to find a continuous selection f of a lower semicontinuous mapping $Cl(\mathrm{conv}(G))$ (see [7] or [14]). Moreover, any selection of $Cl(\mathrm{conv}(G))$ will automatically be a selection of F. The situation is more complicated for the case of nonconvex-valued mappings F.

The notion of the function of nonconvexity of a closed subset of a Banach space was first introduced in [11]. In this paper we consider mappings F whose

1991 *Mathematics Subject Classification.* Primary: 54C60, 54C55. Secondary: 58C05.
Key words and phrases. Multivalued mapping, continuous selections, function of nonconvexity, quasi-lower semicontinuity, fixed-point sets of contractions.
* Supported by the Ministry for Science and Technology of the Republic of Slovenia grant No. J1-7039-0101-95.
** Supported by the RFFI grant No. 96-01-01166a.

values $F(x)$ have some common non-decreasing majorant $\alpha : (0,\infty) \to [0,1)$ for their functions of nonconvexity. In this situation, we have in general, no information about "nonconvexity" of the values $G(x)$ for a lower semicontinuous selection G of F. So we replace the property "F admits a lower semicontinuous selection" by the property "F admits a sufficiently large family of lower semicontinuous selections". The formalisation of the last property leads us to introduce some new classes of non-lower semicontinuous mappings.

We denote by $D(y,r)$ the open ball of radius r, centered at an arbitrary point y of a metric space Y. For any subset $A \subset Y$, we put $D(A,r) = \bigcup\{D(y,r) \mid y \in A\}$ and $D(A,\infty) = Y$. For two multivalued mappings F_1 and F_2 from X into Y we denote by $F_1 \cap F_2$ the mapping $x \mapsto F_1(x) \cap F_2(x)$. For a multivalued mapping $F : X \to Y$ into a metric space Y and for a real-valued mapping $d : X \to (0,\infty)$ we denote by $D(F,d)$ the multivalued mapping $x \mapsto D(F(x),d(x))$. For a closed-valued mapping $F : X \to Y$ into a metric space Y and for a real-valued mapping $\varepsilon : X \to (0,\infty)$ we say that a continuous singlevalued mapping $f : X \to Y$ is an *ε-selection of* F, whenever $\varepsilon(\cdot)$ is a strong majorant of $\mathrm{dist}(f(\cdot),F(\cdot))$, i.e. $\varepsilon(x) > \mathrm{dist}(f(x),F(x))$, for every $x \in X$. We use the term *function* for singlevalued mappings with values from $\mathbb{R}$.

Definition 1.1. Let $\lambda : (0,\infty) \to [1,\infty)$ be any function. Then a closed-valued mapping $F : X \to Y$ to a metric space Y is said to be an *LS_λ-mapping* if for every continuous function $\varepsilon : X \to (0,\infty)$ and every continuous ε-selection f of F, the multivalued mapping $\mathrm{clos}(F \cap D(f,\lambda(\varepsilon)\cdot\varepsilon))$ admits a lower semicontinuous selection.

Each lower semicontinuous mapping F is an LS_1-mapping because the intersection $F \cap D(f,\varepsilon)$ is lower semicontinuous. Clearly, each LS_λ-mapping is an LS_μ-mapping, whenever $\lambda \le \mu$. Next, LS_∞-mappings are exactly those which admit lower semicontinuous selections. We chose the notation LS_λ-mapping as an abbreviation for "mappings, having lower semicontinuous selections with respect to the λ-enlargement of open balls".

Theorem 1.2. *Let $\alpha : (0,\infty) \to [0,1)$ and $\lambda : (0,\infty) \to [1,\infty)$ be any functions such that λ is locally bounded at the origin and $t \mapsto \alpha(\lambda(t)\cdot t)\cdot\lambda(t)$ has a nondecreasing strong majorant $M : (0,\infty) \to [0,1)$. Then every LS_λ-mapping from a paracompact space X into a Banach space Y admits a singlevalued continuous selection, whenever $\alpha(\cdot)$ is a majorant of the set of functions of nonconvexity of values $F(x)$, for every $x \in X$.*

For *constants* α and λ, the hypotheses of Theorem 1.2 are guaranteed by the inequality $\alpha \cdot \lambda < 1$. For the constant λ it suffices to assume that the set of all functions of nonconvexity of values $F(x)$, $x \in X$, has a nondecreasing majorant $\alpha : (0, \infty) \to [0, 1)$. Various weakenings of lower semicontinuity of convex-valued mappings for which a continuous singlevalued selections exist (as in the classical situation) have been intensively studied in the series of papers [1], [2], [5], [6], [9] (see also [10] and [14], § II.3). Most of them are related to the behaviour of a different kind of derived mappings (F', F_0, F_ε) of a given mapping. For the class of so-called *quasi-lower semicontinuous* mappings (see Definition 2.2 below), the derived mapping F' in the sense of Brown [3] is the largest possible lower semicontionuous selection of F. For *convex-valued* quasi l.s.c. mappings and for a *constant* λ, a property somewhat similar to our Definition 1.1 was obtained in [9]. We state the following fact related to Definition 1.1:

Theorem 1.3. *Every quasi-lower semicontinuous mapping of a paracompact space into a complete metric space is an LS_λ-mapping, for each continuous real-valued function $\lambda : (0, \infty) \to (1, \infty)$.*

We derive the following theorem from Theorems 1.2 and 1.3.

Theorem 1.4. *Let $\alpha : (0, \infty) \to [0, 1)$ be a nondecrasing function. Then every quasi-lower semicontinuous mapping F from a paracompact space X into a Banach space admits a singlevalued continuous selection, whenever $\alpha(\cdot)$ is a majorant of the set of functions of nonconvexity of values $F(x)$, for every $x \in X$.*

We list some special cases of Theorem 1.4. For $\alpha(\cdot) \equiv 0$ and any l.s.c. mapping F it yields the Michael convex-valued selection theorem [7]. For $\alpha(\cdot) \equiv q < 1$ and any l.s.c. mapping F we get the Michael paraconvex-valued selection theorem [8]. For $\alpha(\cdot) \equiv 0$ and weakly Hausdorff l.s.c. F (respectively, weakly l.s.c. or quasi l.s.c. F) it gives the DeBlasi-Myjak's (respectively, Przeslawski-Rybinski's or Gutev's) selection theorem [2], [5], [6], [9]. For any nondecreasing function α and for any l.s.c. F it yields a theorem proved earlier by these authors [11], [17]. As an application to the theory of fixed-points of multivalued contractions we can also obtain the following generalization of Ricceri's result [15], in the spirit of the Rybinski paper [16].

Theorem 1.5. *Let X be a paracompact space, Y a Banach space and $X \times Y$ a paracompact space. Suppose that for a multivalued mapping $F : X \times Y \to Y$ and some constants α and γ from $[0, 1)$ the following properties hold:*

(a) *Functions of nonconvexity of all values $F(x, y)$ are less than or equal to α,*
(b) *Each mapping $F(x, \cdot)$ is a γ-contraction,*
(c) *Each mapping $F(\cdot, y)$ is quasi-lower semicontinuous, and*
(d) $\alpha + \gamma < 1$.

Then there exists a singlevalued continuous mapping $f : X \times Y \to Y$ such that for every $x \in X$, the restriction $f(x, \cdot)$ is a retraction onto the fixed-point set of the contraction $F(x, \cdot)$.

2. Preliminaries

We begin by a construction of a function of nonconvexity. For any nonempty closed subset $P \subset Y$ of a Banach space Y and for any open r-ball $D_r \subset Y$, we define the *relative precision* of an approximation of P by elements of D_r as follows:

$$\delta(P, D_r) = \sup \left\{ \frac{\operatorname{dist}(q, P)}{r} \mid q \in \operatorname{conv}(P \cap D_r) \right\}.$$

Clearly, for a convex set P with nonempty intersection $P \bigcap D_r$, the equality $\delta(P, D_r) = 0$ means that this intersection is a convex subset of P.

Definition 2.1. For a nonempty closed subset $P \subset Y$ of a Banach space Y, *the function $\alpha_P(\cdot)$ of nonconvexity of P* associates to each positive number r the following nonnegative number:

$$\alpha_P(r) = \sup\{\delta(P, D_r) \mid D_r \text{ runs over all open } r\text{-balls}\}.$$

Clearly, the identical equality $\alpha_P(\cdot) \equiv 0$ is equivalent to *convexity* of the closed set P. Following Michael [8], the set P is said to be *q-paraconvex*, whenever the number q is a majorant of the function $\alpha_P(\cdot)$. A selection theorem for q-paraconvex valued l.s.c. maps, $q < 1$, was proved in [8]. For a possible substitute of a suitable *function* $q(\cdot)$ instead of the *constant* see [11]. For examples of classes of closed sets with nice functions of nonconvexity see [12], [13], [19].

The notion of quasi lower semicontinuity (respectively, weak lower semicontinuity) of a multivalued mapping was introduced in [5], [6] (respectively, in [9]). Recall, that for a multivalued mapping $F : X \to Y$, the preimage $F^{-1}(A)$, $A \subset Y$, is defined as $\{x \in X \mid F(x) \cap A \neq \emptyset\}$ and for topological spaces X and Y, a mapping F is said to be *lower semicontinuous* if preimages of open sets are open sets.

Definition 2.2. A multivalued mapping $F : X \to Y$ of a topological space X into a metric space (Y, ρ) is said to be *quasi lower semicontinuous* if for every triple $(x, U(x), \varepsilon)$, where $x \in X$, $U(x)$ is a neighborhood of x and $\varepsilon > 0$, there exists a point $q(x) \in U(x)$ such that for every $y \in F(q(x))$, the point x belongs to the interior of the set $F^{-1}(D(y, \varepsilon))$.

Clearly, each l.s.c. map is quasi l.s.c.: it suffices to put $q(x) = x$. For examples of quasi l.s.c., non l.s.c. mappings see [6], [10]. Possibly, one of the simplest examples is given by the mapping $F : X \to [0, \infty)$, $F(x) = [0, l(x)]$, where

$l : X \to [0, \infty)$ is an arbitrary singlevalued locally positive function. We need two Gutev's theorems [6]. Recall that for a multivalued mapping $F : X \to Y$ between topological spaces its *derived* mapping $F' : X \to Y$ is defined by setting $F'(x)$ to be equal to the set of all $y \in F(x)$, for which x belongs to the interior of the preimage (with respect to F) of every neighborhood of y (see [3]).

Theorem 2.3. *Let $F : X \to Y$ be a closed valued quasi lower semicontinuous mapping of a topological space X into a complete metric space (Y, ρ). Then the derived mapping $F' : X \to Y$ is a lower semicontinuous selection of F with nonempty closed values. Moreover, if $G : X \to Y$ is a lower semicontinuous selection of F, then G is also a selection of F'.*

Theorem 2.4. *A mapping $F : X \to Y$ of a topological space X into a complete metric space (Y, ρ) is quasi lower semicontinuous if and only if for every triple $(x, U(x), \varepsilon)$, where $x \in X$, $U(x)$ is a neighborhood of x and $\varepsilon > 0$, there exists a point $q(x) \in U(x)$ such that $F(q(x)) \subset D(F'(x), \varepsilon)$.*

Finally, for each function $M : (0, \infty) \to [0, 1)$ we define the following sequence of functions:

$$M_0(t) \equiv 1, \; M_1(t) = M(t), \; \dots, \; M_{n+1}(t) = M(M_n(t) \cdot t) \cdot M_n(t), \dots$$

Lemma 2.5. *Let $M : (0, \infty) \to [0, 1)$ be a nondecreasing function. Then for every positive τ, the series $\sum_{n=0}^{\infty} M_n(t)$ uniformly converges on the interval $(0, \tau)$.*

3. Proof of Theorem 1.2

Under assumptions of the theorem, let $F : X \to Y$ be a given LS_λ-mapping. Then F is an LS_∞-mapping and, hence, has a lower semicontinuous closed-valued selection, say G. Let $f_0 : X \to Y$ be an arbitrary singlevalued continuous mapping. Then the distance $d(x) = \operatorname{dist}(f_0(x), G(x))$ is an upper semicontinuous real-valued function on the paracompact space X. By the Dowker theorem, the function $d(\cdot)$ has a strong continuous singlevalued majorant, say $\varepsilon : X \to (0, \infty)$. Clearly, f_0 is an ε-selection of F. Now, for every natural number n we put:

$$R_n(x) = M_n(\varepsilon(x)) \cdot \varepsilon(x), \qquad r_n(x) = \lambda(R_n(x)) \cdot R_n(x),$$

where $M : (0, \infty) \to [0, 1)$ is a fixed nondecreasing majorant of the function

$$t \mapsto \alpha(\lambda(t) \cdot t) \cdot \lambda(t)$$

and functions $M_n(\cdot)$ are defined above, before Lemma 2.5. Due to the continuity of the mapping $\varepsilon : X \to (0, \infty)$ and due to Lemma 2.5, for every $x \in X$, there exists its neighborhood $U(x)$ such that the series $\sum_{n=0}^{\infty} R_n(\cdot)$ uniformly converges on $U(x)$. Similarly, the series $\sum_{n=0}^{\infty} r_n(\cdot)$ uniformly converges on $U(x)$, because of local boundedness of the function $\lambda(\cdot)$.

Let us construct a sequence of singlevalued continuous mappings $f_n : X \to Y$ with the properties that for each natural n and for each $x \in X$:

(a_n) $\quad d_n(x) = \operatorname{dist}(f_n(x), F(x)) < R_n(x)$; and

(b_n) $\quad \operatorname{dist}(f_{n+1}(x), f_n(x)) \leq r_n(x)$.

We then see from (b_n) that there exists a pointwise limit $f = \lim_{n\to\infty} f_n$ and that f is a locally (and, hence globally) continuous mapping, due to the local uniform convergence of the series $\sum_{n=0}^{\infty} R_n(\cdot)$ and $\sum_{n=0}^{\infty} r_n(\cdot)$. The closedness of $F(x)$ and inequalities (a_n) imply that f is a selection of F.

So, the mapping f_0 was constructed so that the inequality (a_0) holds. Suppose that for some $n > 0$, we have mappings $f_0, f_1, \dots, f_n$ for which the inequalities $(a_0), (a_1), \dots, (a_n)$ and $(b_0), (b_1), \dots, (b_{n-1})$ hold. By (a_n), the mapping f_n is an R_n-selection of F. Moreover, each nondecreasing mapping $M : (0, \infty) \to [0, 1)$ has a continuous majorant $M_1 : (0, \infty) \to [0, 1)$, i.e. without loss of generality one can assume that $M(\cdot)$ from the hypotheses of the theorem is a continuous function. Hence, the functions $R_n(x) = M_n(\varepsilon(x)) \cdot \varepsilon(x)$ are also continuous and it is possible to use Definition 1.1 of LS_λ-mappings, which directly shows that the mapping

$$x \to Cl(F(x) \cap D(f_n(x), \lambda(R_n(x)) \cdot R_n(x))) = Cl(F(x) \cap D(f_n(x), r_n(x)))$$

admits a lower semicontinuous selection, say G_n. By the classical Michael selection theorem [7], the mapping $Cl(\operatorname{conv}(G_n))$ admits a singlevalued continuous selection, say f_{n+1}. Then

$$f_{n+1}(x) \in Cl(\operatorname{conv}(G_n(x))) \subset Cl(D(f_n(x), r_n(x))),$$

i.e. the inequality (b_n) holds. Now, using Definition 2.1 of the function of nonconvexity for open balls $D(f_n(x), r_n(x))$ and remembering that $\alpha(\lambda(t) \cdot t) \cdot \lambda(t) < M(t)$ for all positive t, we see that:

$$\begin{aligned} \operatorname{dist}(f_{n+1}(x), F(x)) &\leq \alpha_{F(x)}(r_n(x)) \cdot r_n(x) \\ &\leq \alpha(\lambda(R_n(x)) \cdot R_n(x)) \cdot \lambda(R_n(x)) \cdot R_n(x) \\ &< M(R_n(x)) \cdot R_n(x) \\ &= M(M_n(\varepsilon(x)) \cdot \varepsilon(x)) \cdot M_n(\varepsilon(x)) \cdot \varepsilon(x)) \\ &= M_{n+1}(\varepsilon(x)) \cdot \varepsilon(x) = R_{n+1}(x), \end{aligned}$$

i.e. the inequality (a_{n+1}) holds. Theorem 1.2 is thus proved.

Remark. Clearly,

$$\operatorname{dist}(f_0(x), f(x)) \leq \sum_{n=0}^{\infty} r_n(x).$$

4. Proofs of Theorems 1.3-1.5

The initial step of the proofs represents the following lemma, which resulted from our discussions with Gutev.

Lemma 4.1. *Let $F : X \to Y$ be a quasi lower semicontinuous mapping of a topological space X into a complete metric space (Y, ρ), $f : X \to Y$ a singlevalued continuous mapping and $c(\cdot)$ a strong majorant for the distance function $d = \operatorname{dist}(f, F)$. Suppose that the interval-valued mapping $x \mapsto (d(x), c(x))$, $x \in X$, admits a singlevalued continuous selection. Then for every $x \in X$, the intersection $F'(x) \cap D(f(x), c(x))$ is nonempty.*

Proof. Let $s : X \to (0, \infty)$ be a continuous mapping such that $d(x) < s(x) < c(x)$, for every $x \in X$. Pick a point $x \in X$ and put $\varepsilon = (c(x) - s(x))/2$. Let $V = V(x)$ be a neighborhood of x such that the restriction of $s(\cdot)$ onto V is less than $(c(x) + s(x))/2$. Due to the continuity of f, find a neighborhood $U = U(x)$ such that $f(x) \in D(f(z), \varepsilon)$, for every $z \in U$. We can apply Theorem 2.4 to the triple $(x, V \cap U, \varepsilon)$, i.e. we can find a point $q(x) \in V \cap U$ such that

$$F(q(x)) \subset D(F'(x), \varepsilon).$$

By invoking the inequality $d < s$, we see that

$$f(q(x)) \in D(F(q(x)), s(q(x))) \subset D(F'(x), s(q(x)) + \varepsilon).$$

Hence the inequality $s(q(x)) < (c(x) + s(x))/2$, implies that

$$f(x) \in D(f(q(x)), \varepsilon) \subset D(F'(x), s(q(x)) + 2\varepsilon) \subset D(F'(x), c(x)),$$

i.e. the distance between $f(x)$ and $F'(x)$ is less than $c(x)$.

Proof of Theorem 1.3. Let $F : X \to Y$ be a quasi lower semicontinuous mapping of a topological space X into a complete metric space (Y, ρ), $f : X \to Y$ a singlevalued continuous ε-selection of F, for some continuous function $\varepsilon : X \to (0, \infty)$, and $\lambda : (0, \infty) \to (1, \infty)$ a singlevalued continuous function. Then for the (continuous!) strong majorant $c(x) = \lambda(\varepsilon(x)) \cdot \varepsilon(x)$ of the distance function $d(x) = \operatorname{dist}(f(x), F(x))$, there exists an obvious continuous function $s(\cdot)$ which separates $d(\cdot)$ and $c(\cdot)$. Namely, $s(x) = \varepsilon(x)$. Lemma 4.1 shows that the mapping $G = F' \cap D(f, c)$ has nonempty values. But the derived mapping F' is a selection of F. Hence, G is a selection of the mapping $F \cap$

$D(f,c)$. Lower semicontinuity of G follows from the lower semicontinuity of F' (see Theorem 2.3), from continuity of f, and from continuity of $c(\cdot)$. Thus we conclude that the mapping $x \mapsto F(x) \cap D(f(x), \lambda(\varepsilon(x)) \cdot \varepsilon(x))$ admits a lower semicontinuous selection. Theorem 1.3 is thus proved.

Proof of Theorem 1.4. Because of Theorems 1.2 and 1.3 it suffices to check the following simple fact:

Lemma 4.2. *For every nondecreasing function $\alpha : (0,\infty) \to [0,1)$, there exists a continuous function $\lambda : (0,\infty) \to (1,\infty)$ such that the function $\alpha(\lambda(t) \cdot t) \cdot \lambda(t)$ has a nondecreasing strong majorant $M : (0,\infty) \to [0,1)$.*

Proof. It is easy to find a *continuous* nondecreasing majorant $\beta : (0,\infty) \to [0,1)$ of the function $\alpha(\cdot)$ such that $\lim_{t\to\infty} \beta(t) = 1$. Let $\beta(\cdot) < \gamma(\cdot) < M(\cdot) < 1$ and the functions $\gamma(\cdot)$ and $M(\cdot)$ be both continuous and nondecreasing. We claim that $\lambda(\cdot)$ can then be defined as follows:

$$\lambda(t) = \frac{1}{2} \cdot \left(1 + \min\left\{\frac{M(t)}{\gamma(t)}, \frac{\beta^{-1}(\gamma(t))}{t}\right\}\right).$$

Clearly, $\lambda(\cdot)$ is continuous and greater than 1. Moreover,

$$\lambda(t) \cdot t < \beta^{-1}(\gamma(t)),$$
$$\alpha(\lambda(t) \cdot t) \le \beta(\lambda(t) \cdot t) < \gamma(t)$$

and

$$\alpha(\lambda(t) \cdot t) \cdot \lambda(t) < \gamma(t) \cdot \lambda(t) < M(t)$$

due to the choice of $\lambda(t)$. Lemma 4.2 (and hence Theorem 1.4) are thus proved.

Sketch of the proof of Theorem 1.5. First, we refer to [16] for the proof that the hypotheses (b) and (c) together imply the quasi lower semicontinuity of the mapping F in *two* variables and, moreover, of the composition $F(x, h(x,y))$, for each continuous $h : X \times Y \to Y$. Second, (d) implies that $\gamma/(1-\alpha) < 1$ and hence for some numbers $M \in (\alpha, 1)$ and $\lambda > 1$, we have that $\gamma/(1-M) < 1$ and $\gamma \cdot \lambda/(1-M) < 1$.

Now the special case of the selection Theorem 1.2, when α, λ and M are constants, works for the α-paraconvex valued mapping $F_0 = F$ and we can find a selection of F_0, say f_1. Moreover, starting by $f_0(x,y) = y$, we have (see Remark after proof of Theorem 1.2),

$$\operatorname{dist}(f_0(x,y), f_1(x,y)) \le \sum_{n=0}^{\infty} r_n(x,y) = \lambda \cdot \sum_{n=0}^{\infty} M^n \cdot \varepsilon(x,y) = \frac{\lambda}{1-M} \cdot \varepsilon(x,y),$$

for some continuous singlevalued $\varepsilon : X \times Y \to (0,\infty)$.

Put $F_1(x,y) = F_0(x, f_1(x,y))$ and let us estimate the distance between f_1 and F_1:

$$\begin{aligned}\operatorname{dist}(f_1(x,y), F_1(x,y)) &\le H_{\text{dist}}(F_0(x,y), F_0(x, f_1(x,y))) \\ &\le \gamma \cdot \operatorname{dist}(f_0(x,y), f_1(x,y)) \\ &< \gamma \cdot \frac{\mu}{1-M} \cdot \varepsilon(x,y); \qquad \lambda < \mu.\end{aligned}$$

Hence f_1 is an ε_1-selection of F_1 with

$$\varepsilon_1(x,y) = \gamma \cdot \frac{\mu}{1-M} \cdot \varepsilon(x,y).$$

Reapplying Theorem 1.2, we find a selection of F_1, say f_2, with

$$\operatorname{dist}(f_1(x,y), f_2(x,y)) \le \gamma \cdot \frac{\lambda}{1-M} \cdot \varepsilon_1(x,y) < \gamma^2 \cdot \frac{\mu^2}{(1-M)^2} \cdot \varepsilon(x,y).$$

Continuation of this procedure yields the estimate

$$\operatorname{dist}(f_n(x,y), f_{n+1}(x,y)) < q^{n+1} \cdot \varepsilon(x,y), \qquad q = \frac{\gamma \cdot \mu}{1-M}.$$

Having $\gamma \cdot \lambda/(1-M) < 1$, it is clear that we can assume that $\mu > \lambda$ and $q < 1$.

Remark. For the *functions* α and γ of nonconvexity and contractivity one can replace the hypotheses (d), i.e. the numerical inequality $\alpha + \gamma < 1$ by some *functional* expression.

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