Proceedings of the Steklov Institute of Mathematics, Vol. 212, 1996, pp. 3–27.
Translated from Trudy Matematicheskogo Instituta imeni V.A. Steklova, Vol. 212, 1996, pp. 7–32.

On the Softness of the Dranishnikov Resolution[1]

S. M. Ageev, D. Repovš, and E. V. Shchepin

Received April, 1996

1. INTRODUCTION

We understand the *resolution* of the space X to be its representation as a continuous image of some other space $p: \tilde{X} \to X$ which possesses certain additional properties relative to the original.

This work is devoted to the study of the Dranishnikov resolution which represents any compactum in the form of the image of Menger's manifold with the same degree of the local and global connectedness as the compactum itself, under a rather "soft" mapping. The Dranishnikov resolution of any compactum is induced by means of an embedding of the compactum into a Hilbert cube from the *universal Dranishnikov resolution*, i.e., the mapping of Menger's compactum onto the Hilbert cube $d_m: \mu_m \to Q$, which was constructed in [1] and which possesses a large number of properties from which we shall point out the following:

(1) d_m is soft in the following senses: it is polyhedrally m-soft, $(m, m-2)$-soft, $(m-1)$-soft;

(2) d_m^{-1} stably preserves the AE (m)-compacta;

(3) $\dim M_m = m$, and d_m is strictly m-universal.

As is known, the universal Dranishnikov resolution d_m is a kind of a bridge between the theory of Menger manifolds [2] and the theory of Q-manifolds. Note that the resolution d_m has made it possible to formulate and prove the triangulation and stability theorems in the category of Menger manifolds due to its unique property to transfer the LC^{m-1}-compacta into μ^m-manifolds when a preimage is taken.

The Dranishnikov resolution d_m is an important and useful tool in the arsenal of geometrical topology. The original construction and the derivation of its properties are laborious enough to rouse the desire to simplify them.

The construction of the Dranishnikov resolution is based on a certain many-valued retraction of a ball onto a sphere. The main result of this article is to prove the fact that the *type of softness* of the universal resolution d_m coincides with the type of softness of this many-valued retraction. This simplifies considerably the description of the *envelope of softness* of the Dranishnikov resolution which is given below. All the known softness properties of the Dranishnikov resolution are derived from one property, namely, its softness relative to the conservative pairs introduced below. The approach that we propose makes it possible to reveal more softness properties of the Dranishnikov resolution than were revealed by Dranishnikov himself [1].

For the n-dimensional pair (Z, A), any partial lift $\varphi: A \to S^n \times S^n$ of the mapping $\psi: Z \to S^n$ relative to the projection $\mathrm{pr}_2: S^n \times S^n \to S^n$ of the product of n-spheres by the second cofactor

[1]The work was performed under a partial financial support of the International J. Soros Fund (grant NFU 000) and the Russian Foundation for Fundamental Research (project 93-011-1402).

can be extended up to the global lift $\hat{\varphi}: Z \to S^n \times S^n$ of the mapping ψ. However, in this case, the preimage $\hat{\varphi}^{-1}(\mathrm{Diag})$ of the diagonal $\mathrm{Diag} = \{(s,s) \mid s \in S^n\} \subset S^n \times S^n$ must, generally speaking, increase. The n-conservative pairs are precisely the exceptions from this law.

Definition 1.1. The pair (Z, A) is said to be n-conservative if any partial lift $\varphi: A \to S^n \times S^n$ of the mapping $\psi: Z \to S^n$ relative to pr_2 can be extended up to the global lift $\hat{\varphi}: Z \to S^n \times S^n$ of the mapping ψ with the property of preserving the preimage of the diagonal $(\hat{\varphi})^{-1}(\mathrm{Diag}) \subset A$.

We also say that $\hat{\varphi}$ is a Diag-conservative extension of φ up to the lift of the mapping ψ.

In Sec. 3 we describe a sufficiently large reserve on n-conservative pairs (Z, A), namely, either $\dim A \le (n-2)$ and $\dim Z \le n$, or $\dim Z < (n-1)$. More exotic examples of n-conservative pairs are a compact bouquet of a countable number of similar pairs $(I^n, \partial I^n) = (B^n, S^{n-1})$ (the standard pair is H_n), a single-point compactification of a countable number of nonintersecting pairs $(I^n, I^{n-1} \times \{0\})$ consisting of n-cubes and their faces (the standard pair is I_n). Any pair (Z, A) is n-conservative if A is an $(n-1)$-dimensional ANR-compactum. A single-point compactification of a countable number of nonintersecting pairs $(I^n, \partial I^n)$ (the standard pair is J^n) can serve as an example of an n-dimensional not n-conservative pair.

Definition 1.2. The mapping $f: X \to Y$, which is soft relative to all n-dimensional n-conservative pairs (Z, A) is said to be n-conservatively soft.

We shall use the concepts that we have introduced in order to formulate the fundamental theorems.

Theorem 1.3. *The resolution d_m is m-conservatively soft.*

Theorem 1.4. *If the mapping $f: X \to Y$ is m-conservatively soft, then f satisfies properties* (1), (2) *and the following property:*

(4) *the preimage of any conservatively*[2] *closed* equi-LC$^{(m-1)}$*-family* $\{Y_\alpha\}$ *is an* equi-LC^{m-1}*-family.*

The universal Dranishnikov resolution d_m plays, in the category of mappings, a part similar to that played by the universal Menger compactum in the category of spaces. Therefore, by analogy with spaces, certain questions naturally arise concerning the uniqueness and homogeneity of the Dranishnikov resolution. The question concerning the uniqueness remains open, and as to the homogeneity, here the seeming analogy with spaces proved to be violated. The nonhomogeneity of d_m follows from our result concerning the *softness kernel* of the resolution and from the fact that the resolution d_m, being an $(m-1)$-soft mapping, is not, however, m-soft ($\equiv$ an absolute extensor in the dimension m in the category of mappings). By virtue of the finite-dimensional Michael theorem on selection [4], this means that the m-soft kernel $\mathrm{Reg}_m(d_m) = \{m \in M_m \mid$ at the point m the system of fibers $\{d_m^{-1}(q) \mid q \in Q\}$ of the mapping of d_m has the property of equipotential local $(m-1)$-connectedness$\}$ does not coincide with M_m. The invariance of $\mathrm{Reg}_m(d_m)$ under the fiberwise autohomeomorphisms and the following theorem imply the nonhomogeneity of the resolution d_m.

Theorem 1.5. *The m-soft kernel* $\mathrm{Reg}_m(d_m)$ *of the resolution d_m contains the G_δ-set C_m, which is everywhere dense in each of its fibers. Moreover, the restriction $c_m = d_m \restriction C_m$ is an m-soft mapping of C_m onto Q and a strictly m-universal mapping relative to Polish spaces.*

In other words, Chigogidze's resolution [5] is contained in Dranishnikov's resolution as an everywhere dense G_δ-set.

[2] This is how we call a family of sets whose any subfamily has a closed union [3].

The construction of the Dranishnikov resolution given in Sec. 5, just as the original Dranishnikov construction, essentially uses the difficult theorems from the theory of Q-manifolds, namely, the triangulation theorem and the characterization theorem. However, there is reason to hope that we shall be able to do the same without the use of the theory of Q-manifolds.

2. PRINCIPAL CONCEPTS AND FACTS

The problem of extension of partially defined morphisms is of a general-categoric nature. In the TOP category it is known as the problem of continuous extension of the partial mapping $Z \hookleftarrow A \xrightarrow{\varphi} X$ to the whole space Z. The article by Dranishnikov–Dydak from the same volume presents a rich theory ("extension theory") devoted to this problem.

From this theory we shall mention the Kuratovskii relation τ and the Hurewicz–Wallman theorem. The spaces Z and X are related as τ ($Z\tau X$) or X is an absolute extensor of Z ($X \in \mathrm{AE}(Z)$) if any partial mapping $Z \hookleftarrow A \xrightarrow{\varphi} X$ has a global extension $\hat{\varphi}: Z \to X$, $\hat{\varphi}\restriction_A = f$. Detailing this definition for the case where the pair (Z, A) is fixed, we say that $X \in \mathrm{AE}(Z, A)$. The classical theorem [6] of the theory of dimension states that $S^n \in \mathrm{AE}(Z) \iff \dim Z < n$.

In the category of mappings MAP_Y, which have a fixed range Y, the problem of extension of morphisms is known as the problem of extension of a partial lift to the global lift and plays an essential part in the theory of bundles. For the given mapping $f: X \to Y$, the partial lift of the mapping $\psi: Z \to Y$ is the mapping $\varphi: A \to X$ which is defined on the closed subset $A \subset Z$ and which closes diagram (1) to the (square) commutative one:

$$\begin{array}{ccc} A & \xrightarrow{\varphi} & X \\ i\downarrow & & \downarrow f \\ Z & \xrightarrow{\psi} & Y \end{array} \tag{1}$$

The partial lift (1) is extended to the global lift if there exists a global extension $\hat{\varphi}: Z \to X$ of the mapping φ which is the lift of ψ. Thus, the problem of lifting consists in the splitting of the square commutative diagram (1) by the mapping $\hat{\varphi}$ into two triangular commutative diagrams.

Recall that the mapping f is said to be *soft* (*locally soft*) relative to the pair (Z, A) if any partial lift $\varphi: A \to X$ of any mapping $\psi: Z \to Y$ can be extended to the global lifting.

The collection $\mathfrak{S}(f)$ of all pairs (Z, A), which are soft with respect to f, will be called the *softness envelope* of the mapping f.

Note that if $|Y| = 1$, then $(Z, A) \in \mathfrak{S}(f) \iff X \in \mathrm{AE}(Z, A)$. Thus, the problem of extension of lifts is more general than the problem of extension of mappings.

Depending on the softness envelope $\mathfrak{S}(f)$, mappings are divided into Serre bundles and Hurewicz bundles, soft bundles and n-soft bundles, etc.

Definition 2.1. (a) If the softness envelope $\mathfrak{S}(f)$ includes all pairs of the form $(P \times [0,1], P \times \{0\})$, where P is an arbitrary simplex, then f is a Serre bundle.

(b) If $\mathfrak{S}(f)$ includes all pairs of the form $(P \times [0,1], P \times \{0\})$, where P is a simplex of a dimension not larger than n, then f is a Serre n-bundle.

(c) If $\mathfrak{S}(f)$ includes all pairs of the form $(Z \times [0,1], Z \times \{0\})$, where Z is a compactum of dimension not exceeding n, then f is a Hurewicz n-bundle.

Along this line, we can give a more general definition. Suppose that the mapping f is soft (locally soft) with respect to all pairs (Z, A) from a certain class $\mathfrak{C}$. Then f is $\mathfrak{C}$-soft (locally $\mathfrak{C}$-soft). Then we encounter the concepts of (n,k)-soft mappings ($\mathfrak{C} = \{(Z, A) \mid \dim Z \le n,\ \dim A \le k\}$), polyhedrally soft mappings ($\mathfrak{C} = \{(Z, A) \mid Z, A$ are polyhedra, and $\dim Z \le n\}$). The class of (n,n)-soft mappings, or, briefly, n-soft mappings, will be denoted by $\mathfrak{S}_n$ and the class of locally n-soft mappings by $(\mathfrak{L}\mathfrak{S}_n)$.

If the mapping f of the compactum X into a point is n-soft (locally n-soft), then the compactum X itself is called an absolute (neighborhood) extensor in the dimension n and is denoted by AE(n) (ANE(n)). According to Kuratovskii's theorem [7], A[N]E(n)-compacta are characterized by the properties of $(n-1)$-connectedness (C^{n-1}) and a local $(n-1)$-connectedness (LC^{n-1}): $X \in AE(n)$ ($X \in ANE(n)$) $\iff X \in C^{n-1} \cap LC^{n-1}$ ($X \in LC^{n-1}$).

According to Michael's theorem on the selection of many-valued mappings, the local n-softness of the mapping f is equivalent to the equipotential local $(n-1)$-connectedness of the family of all its fibers $\{f^{-1}(y)\} \in$ equi-LC^{n-1}.

The localization of the concept of the equi-LC^n-family is known as a *homotopic regularity* (see [15]). In this work we do not deal with homologic regularity, and therefore, for brevity, we omit the epithet "homotopic" and simply speak about n-regularity. In addition, we change the grading[3] as compared to the preceding works for the sake of the better correspondence of the gradings of softness and regularity.

The mapping $f: X \to Y$ is *n-regular* at the point $x \in X$ if, for any neighborhood $U(x)$, there exist neighborhoods $V(x)$ and $W(f(x))$ such that any mapping $\varphi: S^k \to f^{-1}(y) \cap V(x)$, $y \in W(f(x))$ (otherwise called a k-spheroid) for $k < n$, defined on the boundary of the ball B^{k+1} can be extended to the mapping $\hat{\varphi}: B^{k+1} \to f^{-1}(y) \cap U(x)$ (or, to put it otherwise, is contracted by a $(k+1)$-film).

We denote the set of points of n-regularity of the mapping f by $\mathrm{Reg}_n(f)$ and call it an *n-softness kernel* of the mapping f.

Softness kernels serve as a measure of closeness of mappings to the class of soft mappings. The local n-softness of the mapping $f: X \to Y$ is equivalent to the equality $\mathrm{Reg}_n(f) = X$.

Obviously, the 0-softness kernel coincides with the points of openess of a mapping, the kernels form a decreasing filtration. Here are several more facts.

Proposition 2.2. *The kernel* $\mathrm{Reg}_n(h)$ *of the composition* h *of the mapping* $g \circ f$ *contains the intersection* $\mathrm{Reg}_n(f) \cap f^{-1}(\mathrm{Reg}_n(g))$. *If* f *is an* n*-soft mapping, then* $\mathrm{Reg}_n(h) = f^{-1}(\mathrm{Reg}_n(g))$.

Proposition 2.3. *The kernel* $\mathrm{Reg}_{n+1}(f)$ *of any locally* n*-soft mapping* $f: X \to Y$ *with* LC^n*-fibers contains the preimage* $f^{-1}(Y')$ *of a certain* G_δ*-set* Y'*, which is dense in* Y.

Thus, the kernel $\mathrm{Reg}_{n+1}(f)$ of the mapping described in Proposition 2.3 contains an everywhere dense subset. We shall not use this interesting fact in our work. It should only be pointed out that the Baire theorem on categories plays an essential part in its proof.

We shall need the description of softness kernels in terms of the lifts of mappings. It is known that the n-softness of the mapping f is equivalent to the fact that the standard pairs $J_k = (J_k^+, J_k^-)$ for $k \le n$ belong to the softness envelope of $f: X \to Y$.

Proposition 2.4. *Let* α_k *be a compactifying point* J_k, $k \le n$. *Then* $x_0 \in \mathrm{Reg}_n(f) \iff$

[3] In this work, the n-regularity corresponds to the $(n-1)$-regularity in [15].

any partial lift $\varphi: J_k^- \to X$ of any mapping $\psi: J_k^+ \to Y$ with $\varphi(\alpha_k) = x_0$ can be extended to the neighborhood lift ψ.

Let us consider the behavior of softness kernels for fiberwise products. Recall that the fiberwise product $W = X_f \times_g Z$ of the compacta X and Z with respect to the mappings $f: X \to Y$ and $g: Z \to Y$ is the subset $\{(x, z) \mid f(x) = g(z)\} \subset X \times Z$. The projections of $X \times Z$ onto Z and onto X generate the mappings $f': W \to Z$ and $g': W \to X$, which will be called fiberwise projections parallel to f and g respectively, and will be denoted by $f' \parallel f$, $g' \parallel g$.

Many properties of mappings are inherited by parallel projections. Thus, for instance, the softness envelope $\mathfrak{S}(f)$ is contained in $\mathfrak{S}(f')$. We can establish as easily that $(g')^{-1}\mathrm{Reg}_n(f) \subset \mathrm{Reg}_n(f')$. Now if g was an open mapping, then $\mathrm{Reg}_n(f') = (g')^{-1}(\mathrm{Reg}_n(f))$.

In connection with the formulation and the proof of Theorem 1.5 we introduce the following concept.

Definition 2.5. An *amply n-soft mapping* for $n \geq 0$ is an n-soft mapping $f: X \to Y$ whose softness kernel $\mathrm{Reg}_{n+1}(f)$ contains a G_δ-set C such that

(i) the restriction of f to C is an $(n+1)$-soft mapping onto Y;

(ii) the intersection of C with any fiber $f^{-1}(y)$ is an everywhere dense set in $f^{-1}(y)$.

We say that the G_δ-set C *realizes the ample n-softness of the mapping f* if properties (i), (ii) are satisfied for it.

Proposition 2.6. (a) *The composition of amply n-soft mappings is an ample n-soft mapping.*

(b) *Suppose that $f: X \to Y$ is an amply n-soft mapping and $g: Z \to Y$ is a mapping such that $f^{-1}(y) \subset \mathrm{Reg}_{n+1}(f)$ always implies $g^{-1}(y) \subset \mathrm{Reg}_0(g)$. Then f' is also an amply n-soft mapping.*

In the category G-TOP we encounter the problem of the extension of equivariant mappings and the problem of extension of equivariant partial lifts to global equivariant lifts. Then there appear the concepts of G-$\mathfrak{C}$-soft mappings, where $\mathfrak{C}$ is a certain class of G-pairs. Thus, for instance, there appear G-m-conservatively soft mappings ($\mathfrak{C} = \{(Z, A) \mid$ the pair $((Z/G), (A/G))$ is m-conservatively soft$\}$); G-(n, k)-soft mappings ($\mathfrak{C} = \{(Z, A) \mid \dim(Z/G) \leq n,\ \dim(A/G) \leq k\}$), polyhedrally G-n-soft mappings ($\mathfrak{C} = \{(Z, A) \mid Z/G, A/G$ are polyhedra, and $\dim(Z/G) \leq n\}$).

If the mapping of the G-space X into a point is G-n-soft, then we shall call X an equivariant absolute extensor in the dimension n and denote it by $X \in G\text{-AE}(n)$. In [8] the characterization of G-AE(n)-spaces was obtained in terms of the topological properties of families of H-fixed points $X^H = \{g \cdot x = x$ for all $g \in H\}$: $X \in G\text{-AE}(n) \iff \{X^H \mid H$ is a closed subgroup of $G\} \in$ equi-$\mathrm{LC}^{n-1} \cap C^{n-1}$ and $X \in G\text{-AE}(0)$.

3. PROPERTY OF n-CONSERVATIVENESS

Since, as was mentioned in Introduction, all softness properties of the Dranishnikov resolution d_m are consequences of its softness relative to m-dimensional m-conservative pairs, the properties of m-conservative pairs and those of m-conservative soft mappings must be more thoroughly studied.

First of all, we shall construct a more convenient (for our purposes) than in [1, 9] an n-conservatively soft many-valued retraction of the $(n+1)$-ball B^{n+1} onto its boundary S^n.

Proposition 3.1. *There exist an* ANR*-compactum D_{n+1}, an n-conservatively soft mapping $p_{n+1}: D_{n+1} \to B^{n+1}$, a locally trivial bundle $q_{n+1}: D_{n+1} \to S^n$, and an embedding $i_{n+1}: S_n \to D_{n+1}$ such that*

(1) *q_{n+1} is a retraction, $q_{n+1} \circ i_{n+1} = \mathrm{Id}_{S^n}$;*

(2) *the preimage $p_{n+1}^{-1}(s)$ coincides with $i_{n+1}(s)$ for all $s \in S$;*

(3) *if the equalities $p_{n+1}(d) = p_{n+1}(d')$ and $q_{n+1}(d) = q_{n+1}(d')$ are satisfied for the points $d, d' \in D_{n+1}$, then $d = d'$.*

Remark. (1) The formula $q_{n+1} \circ p_{n+1}^{-1}(b)$, $b \in B^{n+1}$, defines the many-valued retraction of B^{n+1} onto S_n; (2) property (3) is equivalent to the fact that the diagonal product $p_{n+1} \times q_{n+1}: D_{n+1} \to \Delta \times \partial\Delta$ is an embedding.

Proof. Let A_n be a subset $\{(t;x) \doteq (t;x_0,\ldots,x_n) \mid \sum x_i^2 = 1,\ x_0 \le t\}$ in $[-1,1] \times S^n$. The mapping $v_n: A_n \to [-1,1]$, defined by the relation $v_n(x,t) = t$, can be interpreted as the projection of half the cylinder $[-1,1] \times S^n$, resulting from its intersection with the half-space $t - x_0 \ge 0$, onto the generatrix $[-1,1]$. Note that the preimage $v_n^{-1}(t) = A_n \cap (t \times \mathbb{R}^{n+1})$ coincides with the n-sphere S^n from which an open ball with center at the point $\beta = (1;\ldots,0,0)$ is cut out; $v_n^{-1}(-1) = -\beta = \alpha = (-1;\ldots,0,0)$.

We denote by $\widetilde{R}_s$, $s \in S^n$, any turn of the S^n sphere which transfers α into s. The parametric family $R_s = \mathrm{Id}_{[-1,1]} \times \widetilde{R}_s$ of the transformations of the cylinder $[-1,1] \times S^n$, which is, in general, discontinuous, generates the continuous, in the Hausdorff metric, mapping

$$s \in S^n \mapsto R_s(A_n) \in \exp([-1,1] \times S^n).$$

Let us consider the compactum $B_n = \{R_s(A_n) \times s \mid s \in S^n\}$ lying in $[-1,1] \times S^n \times S^n$. Since $R_s(A_n)$ is homeomorphic to A_n for all $s \in S^n$ and continuously depends on s, it follows that the projection of B_n onto the third factor is a locally trivial bundle with a fiber A_n. Therefore B_n is the ANR-compactum.

We denote by $\pi_n: B_n \to [-1,1] \times S^n$ the projection onto the first and third factors and by $\theta_n: B_n \to S^n$ the projection onto the second factor. It is easy to find out that θ_n is a locally trivial bundle with the fiber A_n. As to π_n, we shall prove the following important statement.

Proposition 3.2. *The softness envelope $\mathfrak{S}(\pi_n)$ of the mapping π_n coincides with all n-conservative pairs.*

Proof. It should be noted, first of all, that Diag and the antidiagonal $-$Diag can be transferred into each other by means of a fiberwise autohomeomorphism relative to pr_2. Therefore, when we replace Diag by $-$Diag in Definition 1.1, we get the same class of spaces.

Suppose that π_n is soft with respect to the pair (Z,A) and let $\varphi: A \to S^n \times S^n$ be a lift of the mapping $\psi: Z \to S^n$ relative to the projection $\mathrm{pr}_2: S^n \times S^n \to S^n$, $\mathrm{pr}_2(s_1,s_2) = s_2$. Since A is a zero-set, there exists a numerical function $\xi: Z \to [-1,1]$ with $A = \xi^{-1}(1)$. Then the diagonal mapping $\xi \mid A$ and φ gives a mapping φ from A into B_n, which is a lift of the diagonal mapping $\psi_1: Z \to [-1,1] \times S^n$ of the mappings ξ and ψ. By the hypothesis, there exists a lift $\hat{\varphi}: Z \to B_n$, $\pi_n \circ \hat{\varphi} = \psi_1$, $\hat{\varphi} \restriction A = \varphi_1$. Then the composition of $\hat{\varphi}$ with the projection χ of the compactum B_n onto the second and third factors defines the required Diag-conservative extension of the mapping φ which is the lift of ψ.

Conversely, suppose that we are given an n-conservative pair (Z,A) and a partial lift $\varphi: A \to B_n$ of the mapping $\psi_1: Z \to [-1,1] \times S^n$. Then the composition $\chi \circ \varphi: A \to B_n \to S^n \times S^n$ is a partial lift of the mapping $\xi = \mathrm{pr} \circ \psi: Z \to S^n$ which, by the hypothesis, can be $-$Diag-conservatively extended to the lift $\zeta: Z \to S^n \times S^n$ relative to pr_2. It is clear that ζ is the diagonal product of a certain mapping $\zeta_1: Z \to S^n$ by ξ, and $\zeta_1(z) \neq -\xi(z)$ for $z \notin A$.

Before constructing the global lift $\hat{\varphi}: Z \to B_n$ of the mapping ψ, let us note the following: for $-1 \leq t < 1$, the relation $r_t(x_0, \ldots, x_n) = (\rho_t(x_0), kx_1, kx_2, \ldots, kx_n)$, where $\rho_t(\sigma) = t$ for $\sigma \geq t$, $\rho_t(\sigma) = \sigma$ for $\sigma \leq t$, $k^2 = (1 - (\rho_t(x_0))^2)/(1 - x_0^2)$, correctly defines the retraction $r_t: S^n \backslash \beta \to \{(x_0, \ldots, x_n) \in S^n \mid x_0 \leq t\}$ from the deleted sphere onto the set which is homeomorphic to $v_n^{-1}(t)$.

Then the required global lift $\hat{\varphi}: Z \to B_n$ of the mapping ψ can be constructed according to the following rule:

if $\psi(z) = (1, s)$, then $\hat{\varphi}(z) = (1, \zeta(z)) \in B_n$;

if $\psi(z) = (t, s)$ and $t < 1$, then $\hat{\varphi}(z) = (t, \widetilde{R}_{\xi(z)}(r_t(\zeta_1(z))), \xi(z)) \in B_n$,

where the mapping $r_t: S^n \backslash \beta \to v_n^{-1}(t) \cong \{(x_0, \ldots, x_n) \in S^n \mid x_0 \leq t\}$ which is a retraction, is defined by the relation $r_t(x_0, \ldots, x_n) = (\rho_t(x_0), kx_1, kx_2, \ldots, kx_n)$, $k^2 = (1 - (\rho_t(x_0))^2)/(1 - x_0^2)$, and $\rho_t: [-1, 1) \to [-1, t]$ is a retraction.

Since $v_n^{-1}(1) = S^n$, it follows that B_n contains $\{1\} \times S^n \times S^n$. Therefore, by virtue of the theorem on the addition of ANRs [10, p. 49], $D_{n+1} = B_n \cup (\{1\} \times S_n \times B^{n+1})$ is an ANR-compactum. The projection of D_{n+1} onto the first and third factors $[-1, 1] \times B^{n+1}$ gives the surjection $\pi': D_{n+1} \to [-1, 1] \times S^n \cup \{1\} \times B^{n+1} = \widetilde{B}^{n+1}$ whose image $\widetilde{B}^{n+1}$ is homeomorphic to B_{n+1}. Let $\pi'': \widetilde{B}^{n+1} \to B^{n+1}$ be a homeomorphism, constant on the boundary, i.e., $\pi''(-1, s) = s$ for all $s \in S^n$.

Now, as the required mappings, we set the following: p_{n+1} is the composition $\pi'' \circ \pi': D_{n+1} \to \widetilde{B}^{n+1} \to B^{n+1}$; q_{n+1} is the projection $q_{n+1}: D_{n+1} \to S^n$, $q_{n+1}(t, s_1, s_2) = s_1$, of the compactum D_{n+1} onto the second factor, the embedding $i_{n+1}: S^n \to D_{n+1}$ is defined by the relation $i_{n+1}(s) = (-1, s, s)$. Properties (1) and (3) of Proposition 3.1 are obvious and property (2) follows from the relations $(\pi'')^{-1}(s) = (-1, s)$, $(\pi')^{-1}(-1, s) = (-1, s, s)$, $(p_{n+1})^{-1}(s) = (-1, s, s) = i_{n+1}(s)$.

We shall show that the mapping p_{n+1} is n-conservatively soft. To this end, we pay attention to the fact that by virtue of the construction, $\pi': D_{n+1} \to [-1, 1] \times S^n \cup \{1\} \times B^{n+1} = \widetilde{B}^{n+1}$ over $\{1\} \times B^{n+1}$ is n-soft and over $[-1, 1] \times S^n$ coincides with π_n and is n-conservatively soft. Lemma 3.3(a) completes this part of the proof and, hence, the proof of the whole Proposition 3.1.

Lemma 3.3. *Suppose that $Y = \cup\{Y_i \mid \leq t\}$, f_i is a restriction of $f: X \to Y$ to $X_i = f^{-1}(Y_i)$. If*

(a) *$t = 2$, f_1 is an n-conservatively soft mapping and f_2 is an n-soft mapping, then f is n-conservatively soft;*

(b) *$t \geq 2$, f_i are n-conservatively soft mappings, and the restriction of f_i to Y_j for $i \neq j$ is a homeomorphism, then the mapping f is n-conservatively soft.*

The following proposition demonstrates the *local character* of the property of n-conservativeness.

Proposition 3.4. *Let $\dim Z = k < \infty$. The pair (Z, A) is n-conservative if and only if any partial lift $\varphi: A \to S^n \times S^n$ of the mapping $\psi: A \to S^n$ relative to $\mathrm{pr}_2: S^n \times S^n \to S^n$ is Diag-conservatively extended to the mapping $\tilde{\varphi}: A \cup N(A_0) \to S^n \times S^n$, which is the lift of the mapping ψ, where $N(A_0)$ is a certain closed neighborhood of $A_0 = \varphi^{-1}(\mathrm{Diag})$ in Z.*

We must only prove the sufficiency. Restricting the projection pr_2 to $W = (S^n \times S^n) \backslash \mathrm{Diag}$, we get a locally trivial bundle $\eta: W \to S^n$ with fibers which are homeomorphic to R^n. According to Michael finite-dimensional selection theorem, the mapping η is k-soft.

It follows from the hypothesis that $\tilde{\varphi}$ transfers $F = [A \cup N(A_0)] \setminus \operatorname{Int} N(A_0)$ into W, and therefore there is a lift $\hat{\varphi}$ in the commutative diagram

$$\begin{array}{ccc} F & \xrightarrow{\tilde{\varphi}} & W \\ i\downarrow & & \downarrow\eta \\ Z \setminus \operatorname{Int} N(A_0) & \xrightarrow{\psi} & S^n \end{array}$$

The combining of two mappings $\hat{\varphi}$ and $\tilde{\varphi}$ along their common domain of definition gives the required lift of the mapping ψ which is, at the same time, a Diag-conservative extension of φ.

To conclude this section, we shall give sufficiently extensive conditions that guarantee the n-conservativeness of pairs.

Proposition 3.5. *Let* $\dim A \leq (n-1)$. *Each of the conditions* (α)–(δ) *that follow guarantees the* n*-conservativeness of the pair* (Z, A):

(α) *for any compactum* $A_0 \subset A$ *there exists a neighborhood* U, $A_0 \subset U \subset A$, *such that for any neighborhood* $V, A_0 \subset V \subset U$ *the complement* $V \setminus A_0$ *has zero* $(n-1)$*-dimensional cohomologies* $H^{n-1}(V \setminus A_0, \mathbb{Z}) = 0$ *of Alexandroff–Čech;*

(β) *for any compactum* $A_0 \subset A$, *there exists a neighborhood* U, $A_0 \subset U \subset A$, *such that for any neighborhood* $V, A_0 \subset V \subset U$ *any mapping from* $V \setminus A_0$ *into an* $(n-1)$*-sphere is homotopic to a constant mapping;*

(γ) *for any compactum* $A_0 \subset A$ *there exists a neighborhood* F', $F' \subset Z$, $A_0 \subset \operatorname{Int} F'$, *such that for any other closed neighborhood* F, $F \subset F'$, $A_0 \subset \operatorname{Int} F$, *and any mapping* $f: (F \cap A) \setminus A_0 \to S^{n-1}$ *there exists an extension* $\hat{f}: F \setminus A_0 \to S^{n-1}$, $\hat{f} \mid (F \cap A) \setminus A_0 = f$;

(δ) *for any compactum* $A_0 \subset A$ *there exists a neighborhood* F', $F' \subset Z$, $A_0 \subset \operatorname{Int} F'$, *such that for any other closed neighborhood* F, $F \subset F'$, $A_0 \subset \operatorname{Int} F$, *and any mapping* $f: F \cap A \to B^n$ *with* $f^{-1}(0) = A_0$ *there exists an extension* $\hat{f}: F \to B^n$ *such that* $\hat{f} \mid F \cap A = f$, $\hat{f}^{-1}(0) = A_0$.

Proof. The scheme of the proof of Proposition 3.5 is the following: $(\alpha) \Rightarrow (\beta) \Rightarrow (\gamma) \Rightarrow (\delta) \Rightarrow$ n-conservativeness of the pair (Z, A).

$(\alpha) \Rightarrow (\beta)$. This implication follows from the fact that $\dim(V \setminus A_0) \leq (n-1)$ and $H^{n-1}(V \setminus A_0, \mathbb{Z}) = 0$ entail the homotopy of any mapping from $V \setminus A_0$ into the sphere S^{n-1} to a constant mapping.

$(\beta) \Rightarrow (\gamma)$. Suppose that the neighborhood U, $A \supset U \supset A_0$, is such that any mapping $f: V \setminus A_0 \to S^{n-1}$ is homotopic to the constant ($f \simeq$ const) mapping for all V, $U \supset V \supset A_0$. We take the compactum F' with the condition $A_0 \subset F' \cap A \subset U$ as the required closed neighborhood A_0.

Suppose now that we are given the neighborhood F of the compactum A_0 into F' and the mapping $f: (F \cap A) \setminus A_0 \to S^{n-1}$. We extend f up to the mapping $\zeta: V \setminus A_0 \to S^{n-1}$ to a certain neighborhood V of the compactum A_0 in U. Since ζ is homotopic to a constant mapping by the hypothesis, it follows that $f \simeq$ const too. Hence it follows, according to Borsuk's theorem on the extension of homotopies, that there exists an extension $\hat{f}: F \setminus A_0 \to S^{n-1}$ of the mapping f.

$(\gamma) \Rightarrow (\delta)$. We denote a radial retraction by $r: B^n \setminus \{0\} \to S^{n-1}$, and the mapping $\rho(b) = \|b\|$ by $\rho: B^n \to [0, 1]$.

Suppose that we have to extend the mapping $f: F \cap A \to B^n$ with $f^{-1}(0) = A_0$ to the mapping $\hat{f}: F \to B^n$, $\hat{f} \restriction F \cap A = f$, $\hat{f}^{-1}(0) = A_0$. To do this, we shall consider the compositions $\varphi = r \circ f: (F \cap A)\backslash A_0 \to B^n\backslash\{0\} \to S^{n-1}$ and $\psi = \rho \circ f: F \cap A \to [0,1]$. By virtue of (γ), φ can be extended to the mapping $\zeta: F\backslash A_0 \to S^{n-1}$. The function ψ can be extended to the function $\hat{\psi}: F \to [0,1]$ with the preservation of the preimage of zero by virtue of the following refinement of Urysohn's theorem.

Lemma 3.6. *Suppose that we are given an arbitrary partial mapping $Z \hookleftarrow A \xrightarrow{\varphi} [0,1]$. Then there exists an extension $\hat{\varphi}: Z \to [0,1]$, $\hat{\varphi} \restriction A = \varphi$, such that $\hat{\varphi}^{-1}(0) = \varphi^{-1}(0)$.*

Proof. The required extension $\hat{\varphi}$ is defined by the relation $\hat{\varphi} = 1 - (1 - \psi) \cdot \xi$, where $\psi: Z \to [0,1]$ is an arbitrary extension of φ to Z and the function $\xi: Z \to [0,1]$ is such that $\xi^{-1}(1) = A$.

Let $\hat{\psi}^{-1}(0) = \psi^{-1}(0)$ for the function $\hat{\psi}: F \to [0,1]$. Then the required extension $\hat{f}: F \to B^n$ can be defined by the relation $\hat{f}(z) = 0$ if $z \in A_0$; and $\hat{f}(z) = (\zeta(z), \hat{\psi}(z))$ if $z \in F\backslash A_0$.

$(\delta) \Rightarrow$ the n-conservativeness of the pair (Z, A). Suppose that we are given a partial lift $\varphi: A \to S^n \times S^n$ of the mapping $\psi: Z \to S^n$ relative to pr_2. If we denote $\{(t,s) \in S^n \times S^n \mid s \in S^n$ by W, and the inner product $t \cdot s \geq 0\}$, then the restriction of pr_2 to W is a locally trivial bundle $\tau: W \to S^n$ with a fiber which is homeomorphic to B^n.

Since $\dim A < n$, there exists a closed neighborhood $E \supset A$ such that $\psi' = \psi \restriction E \simeq \mathrm{const}$. Therefore [11] the induced bundle $(\psi')^*(\tau): W_{\mathrm{pr}_2} \times_{\psi'} E \to E$, where $W_{\mathrm{pr}_2} \times_{\psi'} E = \{((t, \psi(e)), e) \mid e \in E,\ t \cdot \psi(e) \geq 0\}$, is a trivial bundle with a fiber B^n.

Let $h: B^n \times E \to W_{\mathrm{pr}_2} \times_{\psi'} E$ be a fiberwise homeomorphism which transfers fiberwise the section $0 \times E$ into the section $\mathrm{Diag}_{\mathrm{pr}_2} \times_{\psi'} E$, i.e., $\mathrm{pr}_2 \circ h = \mathrm{pr}_2$ and $h(0,e) = ((\psi(e), \psi(e)), e)$ for all $e \in E$.

We denote by A_0 the preimage $\varphi^{-1}\mathrm{Diag} \subset A$. Let the neighborhood $Z \supset F' \supset A_0$ satisfy (δ). Then the neighborhood F of the compactum A_0 lying in $E \cap F'$ also satisfies (δ). We assume, without loss of generality, that $\varphi(F) \subset W$. Let us consider the mapping

$$f: F \cap A \xrightarrow{\varphi \times \mathrm{Id}} W_{\mathrm{pr}_2} \times_{\psi'} E \xrightarrow{h^{-1}} B^n \times F \xrightarrow{\mathrm{pr}} B^n.$$

It is obvious that $f^{-1}(0) = A_0$. By virtue of (δ), there exists its extension $\hat{f}: F \to B^n$, $(\hat{f})^{-1}(0) = A_0$, which generates the lift $\hat{\varphi}(z) = h((\hat{f}(z), z))$, $z \in F$, of the mapping $\psi \restriction F$ with the properties $\hat{\varphi} \restriction A = \varphi$, $(\hat{\varphi})^{-1}(\mathrm{Diag}) \subset A$. The proof of the conservativeness of the pair (Z, A) is completed by a reference to Proposition 3.4.

It follows from Proposition 3.5(γ) that we have proved that if the dimension of A is not larger than $(n-2)$ or $\dim(Z) < n$, then the pair (Z, A) is n-conservative. As a corollary of Proposition 3.5(α) we obtain the n-conservativeness of the standard pairs H_n and I_n. Since the $(n-1)$-dimensional ANR-compactum A satisfies condition (α), any compactum Z, that contains A, generates the n-conservative pair (Z, A).

4. PROOF OF THEOREM 1.4

Since the m-conservatively soft mapping f is soft with respect to any one of the pairs enumerated at the end of Sec. 3, it is polyhedrally m-soft, $(m, m-2)$-soft, and $(m-1)$-soft.

We shall show that the softness of the mapping f with respect to the standard pair H_n implies that it stably preserves the AE (n)-compacta and it also preserves property (4) from Introduction. Since an m-conservatively soft mapping is soft with respect to H_m, Theorem 1.4 will be completely proved.

Definition 4.1. (a) The mapping $f: X \to Y$ preserves the AE (n)-compacta (ANE (n)-compacta) if $f^{-1}(Y_0) \in \mathrm{AE}(n)$ ($f^{-1}(Y_0) \in \mathrm{ANE}(n)$) for all AE (n)-compacta (ANE (n)-compacta) $Y_0 \subset Y$.

(b) The mapping $f: X \to Y$ stably preserves the AE (n)-compacta (ANE (n)-compacta) if the product $f \times \mathrm{Id}_Q: X \times Q \to Y \times Q$ of this mapping by the identity mapping of the Hilbert cube Q preserves the AE (n)-compacta (ANE (n)-compacta).

It is easy to find out that the stable preservation of the AE (n)-compacta by the mapping f is equivalent to the fact that the fiberwise product of X by the arbitrary AE (n)-compactum W relative to f and of the arbitrary mapping $g: W \to Y$ is an AE (n)-compactum.

Proposition 4.2. *If the softness envelope $\mathfrak{S}(f)$ of the mapping $f: X \to Y$ contains the standard pairs $H_i = (H_i^+, H_i^-)$ for $i \le n$, then f^{-1} stably preserves the* AE (n)*-compacta.*

Proof. Since when we pass from the mapping f to the projection $f': X_f \times_g W \to W$, which is parallel to it, the softness envelope increases, it suffices to prove in the proposition that $X \in \mathrm{AE}(n)$ in the case where $Y \in \mathrm{AE}(n)$.

Let us assume the contrary, i.e., $X \notin \mathrm{AE}(n) \equiv \mathrm{C}^{n-1} \cap \mathrm{LC}^{n-1}$. Since it is easy to establish that $X \in \mathrm{C}^{n-1}$, it follows that $X \notin \mathrm{LC}^{n-1}$. We assume, without loss of generality, that $X \in \mathrm{LC}^{n-2}$. Since $Y \in \mathrm{AE}(H_i)$ and since H_i belong to the softness envelope $\mathfrak{S}(f)$ for $i \le n$, we can easily find that $X \in \mathrm{AE}(H_i)$ for all $i \le n$. It is stated that the latter inclusions contradict the assumption that $X \notin \mathrm{LC}^{n-1}$.

It is known that the space $V \in \mathrm{LC}^{n-1} \iff$ for any point $v \in V$, for any $\varepsilon > 0$ there exists $\delta > 0$ such that any i-spheroid $\xi: S^i \to N(v, \delta)$, $i < n$, which passes through the point v (i.e., $v \in \xi(S^i)$), is contracted by an $(i+1)$-film $\zeta: B^{i+1} \to N(v, \varepsilon)$.

Since $X \notin \mathrm{LC}^{n-1}$, there exist a point $x_0 \in X$ and a sequence of $(n-1)$-spheroids $\xi_k: S^{n-1} \to X$, which pass through the point x_0, $x_0 \in \xi_k(S^{n-1})$, which converges to this point but does not admit of the contracted of the spheroids ξ_k by the n-films $\zeta_k: B^n \to X$ with the preservation of the convergence $\lim \zeta_k(B^n) = x_0$. Using the sequence $\{\xi_k\}$, we can easily construct the partial mapping $H_n^+ \hookleftarrow H_n^- \xrightarrow{\xi} X$ which, by virtue of $X \in \mathrm{AE}(H_n)$, has a global extension. But this precisely means that almost all ξ_k admit of the contradiction $\zeta_k: B^n \to X$ with the condition $\lim \zeta_k(B^n) = x_0$. We have obtained a contradiction.

It is clear that when the hypothesis of Proposition 4.2 is satisfied, the mapping f does not preserve, in the general case, the equi-LC^{n-1}-families of the compacta. We shall give an important case where the preservation nevertheless takes place (the proof is similar to that of Proposition 4.2).

Proposition 4.3. *If the softness envelope $\mathfrak{S}(f)$ of the mapping $f: X \to Y$ contains the standard pairs $H_i = (H_i^+, H_i^-)$ for $i \leq n$, and the* equi-LC^{n-1}*-family $\{Y_\alpha\}$ of compacta from Y is conservatively closed (and this is equivalent to the property that for any point $y \in Y$ there exists a neighborhood $O(y)$ such that $O(y) \cap Y_\alpha \neq \varnothing \iff y \in Y_\alpha$* (see also footnote 1), *then the preimage $\{f^{-1}(Y_\alpha)\}$ of this family possesses the* equi-LC^{n-1}*-property.*

5. CONSTRUCTING THE m-CONSERVATIVE DRANISHNIKOV RESOLUTION (PROOF OF THEOREM 1.3)

Proposition 5.1. *Suppose that the polyhedron K of dimension k, the natural number m, which is smaller than k, and the number $\varepsilon > 0$ are fixed. Then there exist a triangulation τ of the polyhedron K, an* ANR*-compactum D^m with a certain metric ρ on it, an m-conservatively soft nonexpanding mapping $f_m: D^m \to K$, and an ε-mapping $g_m: D^m \to K^{(m)}$.*

Proof. We choose the triangulation τ of the polyhedron K from the condition $2 \cdot \mathrm{cal}\,(\tau) \cdot (k^2) < \varepsilon$, where we denote by $\mathrm{cal}\,(\tau)$ the largest diameter of the simplex of this triangulation. Let us consider the arbitrary $(n+1)$-simplex $\Delta \in K^{(n+1)}$, $n \geq m$, which is homeomorphic to the ball B^{n+1}. Therefore everything that we have established in Proposition 3.1 for the pair (B^{n+1}, S^n) is also valid for the pair $(\Delta, \partial\Delta)$. In particular, there exist an ANR-compactum D_{n+1}, an n-conservatively soft mapping $p_{n+1}: D_{n+1} \to \Delta$, a locally trivial bundle $q_{n+1}: D_{n+1} \to \partial\Delta$, and an isometric embedding $i_{n+1}: \partial\Delta \to D_{n+1}$ such that

(a) $p_{n+1} \circ i_{n+1} = q_{n+1} \circ i_{n+1} = \mathrm{Id}_{\partial\Delta}$;

(b) the diagonal product $p_{n+1} \times q_{n+1}: D_{n+1} \to \Delta \times \partial\Delta$ is an embedding.

Naturally, the compactum D_{n+1} and all mappings depend on the simplex Δ. If we disjunctively combine all these ANR-compacta over all $(n+1)$-simpleces $\Delta \in K^{n+1}$ and then carry out the identification with respect to the images $i^{n+1}(\partial\Delta)$ of the boundaries of the simpleces Δ, then we get the ANR-compactum D_{n+1}, the embedding $i^n_{n+1}: K^{(n)} \to D_{n+1}$, the mapping $p^n_{n+1}: D^n_{n+1} \to K^{(n+1)}$ and $q^n_{n+1}: D^n_{n+1} \to K^n$, for which the following properties are valid:

(α) $q^n_{n+1} \circ i^n_{n+1} = p^n_{n+1} \circ i^n_{n+1} = \mathrm{Id}_{K^{(n)}}$;

(β) the diagonal product $p^n_{n+1} \times q^n_{n+1}: D^n_{n+1} \to K^{(n+1)} \times K^{(n)}$ is an embedding;

(γ) $q^n_{n+1}[(p^n_{n+1})^{-1}(\Delta)] \subset \partial\Delta$ for all simpleces Δ of the triangulation τ.

The restriction of p^n_{n+1} to the complete preimage $(p^n_{n+1})^{-1}(\Delta)$, $\Delta \in K^{(n+1)}$, coincides with p_{n+1} and, consequently, is n-conservative; now the restriction of p^n_{n+1} to $(p^n_{n+1})^{-1}(K^{(n)})$ is a homeomorphism. Using Lemma 3.3(b), we come to the conclusion that p^n_{n+1} is n-conservative.

The constructions given above can be complemented by the operation of fiberwise multiplication to the Pascal "triangle."

$$
\begin{array}{ccccccccccc}
K & & & & & & & & & & \\
p_k^{k-1}\uparrow & & & & & & & & & & \\
D_k^{k-1} & \xrightarrow{q_k^{k-1}} & K^{(k-1)} & & & & & & & & \\
p_k^{k-2}\uparrow & & p_{k-1}^{k-2}\uparrow & \nwarrow & & & & & & & \\
D_k^{k-2} & \xrightarrow{q_k^{k-2}} & D_{k-1}^{k-2} & \xrightarrow{q_{k-1}^{k-2}} & K^{(k-2)} & & & & & & \\
p_k^{k-3}\uparrow & & p_{k-1}^{k-3}\uparrow & & p_{k-2}^{k-3}\uparrow & \nwarrow & & & & & \\
\vdots & & \vdots & & \vdots & & & & & & \\
p_k^{m+1}\uparrow & & p_{k-1}^{m+1}\uparrow & & p_{k-2}^{m+1}\uparrow & & & \nwarrow & & & \\
D_k^{m+1} & \xrightarrow{q_k^{m+1}} & D_{k-1}^{m+1} & \xrightarrow{q_{k-1}^{m+1}} & D_{k-2}^{m+1} & \xrightarrow{q_{k-2}^{m+1}} & \ldots & \xrightarrow{q_{m+2}^{m+1}} & K^{(m+1)} & & \\
p_k^{m}\uparrow & & p_{k-1}^{m}\uparrow & & p_{k-2}^{m}\uparrow & & & & p_{m+1}^{m}\uparrow & \nwarrow & \\
D_k^{m} & \xrightarrow{q_k^{m}} & D_{k-1}^{m} & \xrightarrow{q_{k-1}^{m}} & D_{k-2}^{m} & \xrightarrow{q_{k-2}^{m}} & \ldots & \xrightarrow{q_{m+2}^{m}} & D_{m+1}^{m} & \xrightarrow{q_{m+1}^{m}} & K^{(m)}
\end{array}
$$

To this end, we set the compactum D_{t+1}^{s-1} for $t-s+2 \geq 2$ equal to the fiberwise product of the compacta D_{t+1}^{s} and D_t^{s-1} already constructed relative to the mappings q_{t+1}^{s} and p_t^{s-1}.

$$
\begin{array}{ccc}
D_{t+1}^{s-1} & \xrightarrow{q_{t+1}^{s-1}} & D_t^{s-1} \\
p_{t+1}^{s-1}\downarrow & & \downarrow p_t^{s-1} \\
D_{t+1}^{s} & \xrightarrow{q_{t+1}^{s}} & D_t^{s}
\end{array}
$$

We denote the projections parallel to the mappings q_{t+1}^{s} and p_t^{s-1} by q_{t+1}^{s-1} and p_{t+1}^{s-1}. We shall now introduce into consideration the compositions of the mappings along the legs of the Pascal "triangle":

$$f_m \colon D^m = D_k^m \xrightarrow{p_k^m} D_k^{m+1} \xrightarrow{p_k^{m+1}} D_k^{m+2} \xrightarrow{p_k^{m+2}} \ldots \xrightarrow{p_k^{k-2}} D_k^{k-1} \xrightarrow{p_k^{k-1}} K,$$

$$g_m \colon D^m \xrightarrow{q_k^m} D_{k-1}^m \xrightarrow{q_{k-1}^m} D_{k-2}^m \xrightarrow{q_{k-2}^m} \ldots \xrightarrow{q_{m+2}^m} D_{m+1}^m \xrightarrow{q_{m+1}^m} K^{(m)}.$$

Proposition 5.2. *The mapping f_m is m-conservatively soft.*

Proof. It is easy to find out that the composition of m-conservatively soft mappings as well as the projection of a fiberwise product, which is parallel to an m-conservatively soft mapping, is an m-conservatively soft mapping. Hence follows an m-conservative softness of f_m.

The polyhedron D^m is naturally homeomorphic to the set $\{a = (a_k, a_{k-1}, \ldots, a_{m+1}) \in D_k^{k-1} \times \ldots \times D_{m+1}^m \mid q_i^{i-1}(a_i) = p_{i-1}^{i-2}(a_{i-1}),\ i = k, \ldots, m+2\}$, and the values of the mappings f_m and q_m at the point a are equal to $p_k^{k-1}(a_k)$ and $q_{m+1}^m(a_{m+1})$ respectively. This remark and (γ) yield the following important property of the mappings f_m and q_m.

(δ) $q_m \circ f_m^{-1}(\Delta) \subset \Delta^{(m)}$ for all simpleces Δ of the triangulation τ.

We define the pseudometric on D^m by the relation

$$\rho(a,a') = \sum_{t=m+1}^{k} d(p_t^{t-1}(a_t), p_t^{t-1}(a'_t)) + \sum_{t=m+1}^{k} d(q_t^{t-1}(a_t), q_t^{t-1}(a'_t)),$$

where $a = (a_k, a_{k-1}, \dots, a_{m+1})$ and $a' = (a'_k, a'_{k-1}, \dots, a'_{m+1})$ are points from D^m and d is a metric on K. It follows from (β) that if $\rho(a,a') = 0$, then $a = a'$. Therefore ρ is a metric on D^m. It can be immediately established that the topology τ_ρ on D^m, generated by ρ, is not weaker than the original topology τ. From this and from the compactness of D^m it follows that the metric ρ is consistent with the topology τ.

Since $f_m(a) = p_k^{k-1}(a_k)$, the mapping f_m is a nonexpanding mapping.

We shall show that g_m is an ε-mapping. To do this, we must establish the inequality $\rho(a,a') < \varepsilon$ if $q_{m+1}^m(a_{m+1})$ and $q_{m+1}^m(a'_{m+1})$ coincide. It is easy to get the following estimate from the inclusion (δ).

Lemma 5.3. *Let $b, b' \in D^m$, $k \geq t > m$.*

If $d(q_t^{t-1}(b_t), q_t^{t-1}(b'_t)) < \delta$, then $d(p(q_t^{t-1}(b_t), p_t^{t-1}(b'_t)) < \delta + 2 \cdot \mathrm{cal}\,(\tau)$.

Using repeatedly this lemma for $m+1, \dots, t$, we find for $a, a' \in D^m$ that

$$d(q_{t+1}^t(a_{t+1}), q_{t+1}^t(a'_{t+1})) = d(p_{t+1}^t(a_{t+1}), p_{t+1}^t(a'_{t+1})) < 2 \cdot \mathrm{cal}\,(\tau) \cdot (t-m).$$

Consequently,

$$\rho(a,a') < 2 \cdot \mathrm{cal}\,(\tau) \cdot (1 + 2 + \ldots + (k-m)) < 2 \cdot \mathrm{cal}\,(\tau) \cdot (k^2) < \varepsilon.$$

In order to prove $D^m \in \mathrm{ANR}$, we represent D^m in the form of a finite union of sets of the form

$$\mathfrak{D}_\sigma = [\Delta_k, \Delta_{k-1}, \dots, \Delta_m] = \{a \in D_m \mid q_i^{i-1}(a_i) \in \Delta_{i-1} \text{ for } k \geq i > m,\ p_k^{k-1}(a_k) \in \Delta_k\},$$

where $\sigma = \{\Delta_k \supset \Delta_{k-1} \supset \ldots \supset \Delta_m\}$ is a decreasing sequence of simpleces of the triangulation τ which satisfy the inequalities $\dim \Delta_i \leq i$. Since all $q_i \restriction (q_{i-1})^{-1}(\Delta_{i-1})$ are trivial bundles with certain fibers $F_i \in \mathrm{ANR}$, it is easy to establish that $\mathfrak{D}_\sigma$ is homeomorphic to the product Δ_m of the ANR-compacta F_i for which $\Delta_{i-1} \neq \Delta_i$, i.e., $\mathfrak{D}_\sigma \in \mathrm{ANR}$. Since the intersection of a finite number of $\mathfrak{D}_\sigma$ is a set of the same form (and, hence, is an ANR), it follows, according to the theorem on the union of ANRs that $D^m \in \mathrm{ANR}$.

We have thus completed the proof of Proposition 5.1.

Most likely, the compactum D^m is a polyhedron [9, p. 124]. However, we cannot consider this fact to be strictly established. Now if we do not make an effort to prove it, then we have to use the laborious theorems from the theory of Q-manifolds [12].

Edwards' theorem. *If the compactum X is an* ANR, *then the product of X by the Hilbert cube Q is a Q-manifold.*

Chapman's theorem. *Any Q-manifold X is a product of a certain polyhedron P by Q.*

Proposition 5.4. *For any polyhedron K, any metric ρ on $K \times Q$, and the number $\varepsilon > 0$ there exist*

(1) *a polyhedron P and a metric d on $P \times Q$,*

(2) *an m-dimensional polyhedron L and a number δ,*

(3) *mappings* $\alpha: P \times Q \to K \times Q$ *and* $\beta: P \times Q \to L$ *such that*

(4) α *is an m-conservatively soft mapping,*

(5) β *is a* δ*-mapping (i.e.,* $\operatorname{diam} \beta^{-1}(*) < \delta$*),*

(6) α *transfers any set* $A \subset P \times Q$ *with a diameter smaller than* δ *into a set with a diameter smaller than* ε.

Proof. We represent the cube $Q = I^N \times Q'$ so that the natural projection $p: K \times Q \to K \times I^N = K_1$ is an $(\varepsilon/8)$-mapping.

We choose the triangulation τ of the polyhedron K_1 for which there exist an ANR-compactum D^m, an m-conservatively soft nonexpanding mapping $f_m: D^m \to K_1$, and an $(\varepsilon/8)$-mapping g_m: $D^m \to K_1^{(m)} = L$.

On the Q-manifold $D^m \times Q$ we consider a metric equal to the product of the metric on D^m and the metric on Q relative to which Q has a diameter smaller than $\varepsilon/8$. According to Chapman's theorem, there exists a polyhedron P whose product by Q is homeomorphic to $D^m \times Q$. Let $h: P \times Q \to D^m \times Q$ be the corresponding homeomorphism and the metric d on $P \times Q$ be that relative to which h is an isometry.

We set the mapping α equal to the composition $P \times Q \xrightarrow{h} D^m \times Q \xrightarrow{f_m \times \mathrm{Id}} K_1 \times Q'$ and β equal to the composition $P \times Q \xrightarrow{h} D^m \times Q \xrightarrow{\mathrm{pr}} D^m \xrightarrow{g_m} K_1^m = L$.

Obviously, α satisfies property (4). Taking $\delta = 3 \cdot \varepsilon/8$, we verify the other properties (5), (6): $\operatorname{diam}((g_m \circ \mathrm{pr} \circ h)^{-1}(*)) = \operatorname{diam}((g_m \circ \mathrm{pr})^{-1}(*)) \le \operatorname{diam}((g_m)^{-1}(*)) + 2 \cdot \varepsilon/8 \le 3 \cdot \varepsilon/8 = \delta$.

Let $A \subset P \times Q$ and $\operatorname{diam} A < \delta$. Then $\operatorname{diam}(B) \le \delta + 2 \cdot \varepsilon/8 \le 5 \cdot \varepsilon/8$, where $B = \mathrm{pr}(hA)$. Then $\operatorname{diam}(\alpha A) \le \operatorname{diam}(f_m B \times Q) \le 5 \cdot \varepsilon/8 + 2 \cdot \varepsilon/8 < \varepsilon$.

The commutative diagrams (2_t) for $t = 1, 2, 3, \ldots$

$$\begin{array}{ccc} X_{t+1} & \xrightarrow{\theta_t} & X_t \\ f_{t+1} \downarrow & & \downarrow f_t \\ Y_{t+1} & \xrightarrow{\sigma_t} & Y_t \end{array} \tag{2_t}$$

generate a mapping of limits of the inverse spectra $f: \varprojlim \{X_t, \theta_t\} \to \varprojlim \{Y_t, \sigma_t\}$. Generally speaking, the n-softness (n-conservative softness, etc.) of all mappings f_t, σ_t, θ_t does not imply the corresponding softness of the mapping f. However, if all (2_t) diagrams are, say, n-soft in the sense of [13], then f is also n-soft. Since it is necessary to track more carefully the softness properties when we pass to the limits of inverse spectra, we introduce the following definition of softness relative to a pair.

Definition 5.5. We say that the commutative diagram (2_t) is *soft relative to the pair* (Z, A) if the characteristic mapping of this diagram $g_t: X_{t+1} \to (Y_{t+1})_{\sigma_t} \times_{f_t} X_t = Z_t$ into the fiberwise product Z_t, defined by the relation $g_t(x) = (f_{t+1}(x), \theta_t(x)) \in Z_t$, is soft relative to the same pair (Z, A).

A commutative diagram is said to be *n-conservatively soft* (*n-soft*, etc.) if the characteristic mapping is n-conservatively soft (n-soft, etc.).

Let us generalize the result of [13] to a more general situation.

Proposition 5.6. *If all commutative diagrams* (2_t) *are soft relative to the pair* (Z, A) *and the mapping* f_1 *is soft relative to* (Z, A), *then the limit mapping* $f: \varprojlim\{X_t, \theta_t\} \to \varprojlim\{Y_t, \sigma_t\}$ *is also soft relative to* (Z, A).

Proposition 5.7. *Suppose that the mapping* θ_t *is represented in the form of the composition* $X_{t+1} \xrightarrow{\theta''_t} Z_t \xrightarrow{\theta'_t} X_t$ *and the compactum* X_{t+1} *is a fiberwise product of the compacta* Z_t *and* Y_{t+1} *relative to the mappings* $h_t = f_t \circ \theta'_t$ *and* σ_t, *with* $\theta''_t \parallel \sigma_t$, $f_{t+1} \parallel h_t$. *If all mappings* f_t, θ'_t, σ_t *are soft relative to the pair* (Z, A) *for all* $t \geq 1$, *then the limit mapping* f *is soft relative to the same pair.*

The Dranishnikov resolution is obtained as a limit mapping generated by the commutative diagrams (3_t),

$$\begin{array}{ccc} K_{t+1} \times Q & \xrightarrow{\theta_t} & K_t \times Q \\ e_{t+1} \downarrow & & \downarrow e_t \\ I^{t+1} & \xrightarrow{\sigma_t} & I^t \end{array} \qquad (3_t)$$

which satisfy Proposition 5.5, where K_t is a polyhedron, I^t is a cube of dimension t, and σ_t is a projection along the last factor I.

We shall successively construct these diagrams (3_t). For $t = 1$, we take the segment I^1 as the polyhedrona K_t, set the mapping $e_t: K_t \times Q \to I^t$ equal to the projection along Q, and fix the metric ρ_t on $K_t \times Q$.

Suppose that we have already constructed the mappings $e_s: K_s \times Q \to I^t$, $\theta_s: K_{s+1} \times Q \to K_s \times Q$ and the metrics ρ_s on $K_s \times Q$ for all $s \leq t$. We choose the number $\varepsilon = \varepsilon_t$ from the following condition: $\operatorname{diam}[\theta_s \circ \ldots \circ \theta_{t-1}(A)] < 2^{-t}$ for all $s < t$ and all $A \subset K_t \times Q$ with $\operatorname{diam} A < \varepsilon_t$. For the number ε, the polyhedron K_t, and the metric ρ_t on $K_t \times Q$ we choose the mappings $\alpha_t: P_t \times Q \to K_t \times Q$ and $\beta_t: P_t \times Q \to L_t$, $\dim L_t \leq m$, as well as the metric ρ'_{t+1} on $P_t \times Q$, the number $\delta_{t+1} > 0$ proposed in Proposition 5.4. We denote by W_{t+1} the fiberwise product of the compactum $(P_t \times Q) \times T$, where T is a $(2m+1)$-cube I^{2m+1}, by the cube I^{t+1} relative to the composition

$$h_t: P_t \times Q \times T \xrightarrow{\text{pr}} P_t \times Q \xrightarrow{\alpha_t} K_t \times Q \xrightarrow{e_t} I^t \text{ and } \sigma_t: I^{t+1} \to I^t.$$

We denote by $\gamma_{t+1}: W_t \to P_t \times Q \times T$ and $e_{t+1}: W_{t+1} \to I^{t+1}$ the projections which are parallel to σ_t and h_t respectively. It is obvious that the fiberwise product W_{t+1} coincides with $P_t \times Q \times T \times I$ (i.e., with the product of polyhedron $K_{t+1} = P_t \times T \times I$ by Q) and the mapping γ_t is a projection along I and, consequently, nonexpandable, if we consider on $K_{t+1} \times Q$ the product of the metric ρ'_{t+1} by the metric on I. Consequently, if e_t was an m-conservatively soft mapping, then the mappings h_t and e_{t+1} (since $e_{t+1} \parallel h_t$) and the composition $\theta_t = h_t \circ \gamma_{t+1}$ are m-conservatively soft. It follows immediately that the characteristic mapping $g_t: K_{t+1} \times Q \to (I^{t+1})_{\sigma_t} \times_{e_t} (K_t \times Q)$ of the diagram (3_t) is m-conservatively soft.

According to Proposition 5.6, the limit mapping

$$d_m: M_m = \varprojlim \{K_t \times Q, \theta_t\} \to \varprojlim \{I^t, \sigma_t\} = Q,$$

generated by the diagrams $\mathfrak{D}'_t$, is m-conservatively soft.

Since the estimate

$$\operatorname{diam}(\theta_s \circ \cdots \circ \theta_t)(r_{t+1}^{-1})(*) < 2^{-t} \text{ for all } s < t,$$

is valid for the mapping

$$r_t: K_{t+1} \times Q \xrightarrow{\gamma_{t+1}} P_t \times Q \times T \xrightarrow{\text{pr}} P_t \times Q \xrightarrow{\beta_t} L_t$$

onto the m-dimensional polyhedron L_t, the limit spectrum M_m admits of arbitrarily small mappings into m-dimensional polyhedra. Therefore $\dim M_m \leq m$.

Since the projections θ_t coincide with the compositions $\gamma_t \circ \text{pr} \circ \alpha_t$, and the projection pr is along the cube T of a sufficiently large dimension, it is easy to derive a strong m-universality of the resolution d_m, i.e., for any $\varepsilon > 0$ and any mapping $\varphi: Z \to M_m$ there exists an embedding $\tilde{\varphi}: Z \to M_m$ with properties $(\varphi, \tilde{\varphi}) < \varepsilon$, $\varphi \circ d_m = \tilde{\varphi} \circ d_m$.

6. INTERMEDIATE SOFTNESS AND RELATIONSHIPS BETWEEN DIFFERENT CLASSES OF MAPPINGS

The class of n-conservatively soft mappings lies between the classes of n- and $(n-1)$-soft mappings. Therefore mappings of this class are also called $(n-1/2)$-soft and are denoted by $\mathfrak{S}_{(n-1/2)}$.

The Dranishnikov resolution d_n is an example of $(n-1/2)$-soft but not an n-soft mapping. There exists a 0-soft mapping of AE (1)-compacta which do not have sections over the arcs. Thus, the classes of $(n-1/2)$- and $(n-1)$-soft mappings are identical. How large is the difference between them? We pointed out earlier that the softness envelope of an $(n-1/2)$-soft mapping contained pairs I_k for $k \leq n$. What does this property entail?

Definition 6.1. We say that the mapping $f: X \to Y$ lifts small homotopies of n-polyhedra to small homotopies if, for any $\varepsilon > 0$, there exists $\delta > 0$ such that any homotopy $\psi: P \times I \to Y$ with a diameter smaller than δ of the polyhedron P of a dimension not exceeding n, partially lifted to the mapping $\varphi: P \times \{0\} \to X$, $\operatorname{diam} \varphi < \delta$ is lifted to the homotopy $\tilde{\varphi}: P \times I \to X$ with $\operatorname{diam} \tilde{\varphi} < \varepsilon$.

Proposition 6.2. *The mapping $f: X \to Y$ which is an $(n-1)$-bundle, lifts small homotopies of $(n-1)$-polyhedra to small homotopies $\Longleftrightarrow$ the softness envelope $\mathfrak{S}(f)$ contains the pairs I_k for $k \leq n$.*

Proposition 6.3. *If the softness envelope $\mathfrak{S}(f)$ of the mapping $f: X \to Y$ between the* ANE (1)-*compacta contains the pairs I_k for $k \leq n$, and all fibers $f^{-1}(y) \in$ AE $(n-1)$, then f is $(n-1)$-soft.*

Proof. Having assumed the contrary (f is not $(n-1)$-soft), we get a family of fibers without the property equi-LC^{n-2}. In turn, this means that

(1) there exists a convergent sequence $y_k \to y_0 \in Y$;

(2) there exists $x_0 \in f^{-1}(y_0)$ and there exists a sequence of $(n-2)$-spheroids $\varphi_k: S^{n-2} \to f^{-1}(y_k)$ which converge to the point x_0, but do not admit of the contraction of the spheroids φ_k by $(m-1)$-films $\zeta_k: B^{n-1} \to f^{-1}(y_k)$ with the preservation of the convergence $\lim \zeta_k(B^{n-1}) = x_0$.

Since $Y \in \text{LC}^0$, there are paths $s_k: [0,1] \to Y$, $s_k(0) = y_0$, $s_k(1) = y_k$, which converge to y_0. The small homotopies $\Phi_k: S^{n-2} \times [0,1] \xrightarrow{\text{pr}_2} [0,1] \xrightarrow{s_k} Y$ are lifted to the small homotopies $\tilde{\Phi}_k: S^{n-2} \times [0,1] \to X$, $f \circ \tilde{\Phi}_k = \Phi_k$, $\tilde{\Phi}_k \restriction S^{n-2} \times \{1\} = \varphi_k$, and, consequently, $\lim(\operatorname{Im} \tilde{\Phi}_k) = x_0$.

All $(n-2)$-spheroids $\psi_k = \tilde{\Phi}_k \restriction S^{n-2} \times \{0\}$ lie in the fiber $f^{-1}(y_0)$ and converge to x_0. Since $f^{-1}(y_0) \in \mathrm{LC}^{n-2}$, there exist $(n-1)$-films $\hat{\psi}_k: B^{n-1} \to f^{-1}(y_0)$, which contract ψ_k and converge to x_0.

Combining the homotopies $\tilde{\Phi}_k$ and the $(n-1)$-films $\tilde{\psi}_k$ along their common domain of definition, we get m-films $\xi_k: B^{n-1} \to X$, $\xi_k \restriction S^{n-2} = \varphi_k$, $\mathrm{Im}\,(f \circ \xi_k) = \mathrm{Im}\, s_k$, converging to x_0. Now we contract s_k along itself to the point y_k leaving it fixed. In this way it is easy to construct contractions $H_t^k: B^{n-1} \to Y$, $H_0^k = f \circ \xi_k$, $H_1^k = \{y_k\}$, $H_t^k \restriction S^{n-2} = f \circ \varphi_k$, $\mathrm{Im}\,(H_t^k) \subset \mathrm{Im}\, s_k$, which also converge to the point y_0. We lift these small homotopies of an $(n-1)$-ball to homotopies $\tilde{H}_t^k: B^{n-1} \to X$, $f \circ \tilde{H}_t^k = H_t^k$, $\tilde{H}_t^k \restriction S^{n-2} = \varphi_k$, converging to x_0. As a result we have $(n-1)$-films $\tilde{H}_1^k: B^{n-1} \to f^{-1}(y_k)$, $\tilde{H}_1^k \restriction S^{n-2} = \varphi_k$, which contract $(n-2)$-spheroids φ_k and converge to the point x_0. We have thus obtained a contradiction with the assumption that we have made. Consequently, f is $(n-1)$-soft.

Since the fibers of the $(n-1/2)$-soft mapping are $\mathrm{AE}\,(n)$-compacta, the proposition we have proved implies the inclusions of the following classes of mappings between $\mathrm{AE}\,(1)$-compacta:

$$\mathfrak{S}_{n-1/2} \subset \{f \mid I_k \in \mathfrak{S}(f) \text{ for } k \le n,\ f^{-1}(y) \in \mathrm{AE}\,(n)\} \subset \mathfrak{S}_{n-1}.$$

Let us show that between the second and the third class of mappings there is a class of mappings which stably preserve the $\mathrm{AE}\,(n)$-compacta.

Proposition 6.4. *If the softness envelope $\mathfrak{S}(f)$ of the mapping $f: X \to Y$ between* $\mathrm{ANE}\,(1)$*-compacta contains standard pairs I_i for $i \le n$, and all fibers $f^{-1}(y) \in \mathrm{AE}\,(n)$, then f stably preserves the* $\mathrm{AE}\,(n)$*-compacta.*

Proof. Just as in Proposition 5.2, it suffices to establish that $X \in \mathrm{ANE}\,(n)$ follows from $Y \in \mathrm{AE}\,(n)$.

Assuming the contrary, we get a point $x_0 \in X$, a sequence of $(n-1)$-spheroids $\varphi_k: S^{n-1} \to X$, which converge to the point x_0 but do not admit of the contraction of the spheroids φ_k by m-films $\zeta_k: B^n \to X$ with the preservation of the convergence $\lim \zeta_k(B^n) = x_0$.

Since $Y \in \mathrm{LC}^{n-1} \cap \mathrm{C}^{n-1}$, the spheroids $f \circ \varphi_k$ are contracted by n-films $\psi_k: B^n \to Y$, $\psi_k \restriction S^{n-1} = f \circ \varphi_k$, $\psi_k(0) = f(x_0)$, such that we have the convergence $\lim \psi_k(B^n) = f(x_0)$.

We use the fact that $I_n \in \mathfrak{S}(f)$ and cover the small homotopies, defined by n-films ψ_k, by small homotopies $\tilde{\psi}_k: S^{n-1} \times [0,1] \to X$ such that $f \circ \tilde{\psi}_k(s,t) = \tilde{\psi}_k(t \cdot s)$ and $\tilde{\psi}_k \restriction S^{n-1} \times \{1\} = \varphi_k$. It is clear that $(n-1)$-spheroid $\xi_k = \tilde{\psi} \restriction S^{n-1} \times \{0\}$ is contained in the fiber $f^{-1}(y_0)$, where $y_0 = f(x_0)$ and $\lim \xi_k(S^{n-1}) = x_0$.

Since $f^{-1}(y_0) \in \mathrm{AE}\,(n)$, by the hypothesis, the spheroids ξ_k are contracted by the films $\hat{\xi}_k: B^n \to f^{-1}(y_0)$ with the preservation of the convergence $\lim \hat{\xi}_k(B^n) = x_0$.

Combining the homotopy $\tilde{\psi}_k$ and the n-film $\hat{\xi}_k$ along their common domain of definition, we get n-films $\zeta_k: B^n \to X$, $\zeta_k \restriction S^{n-1} = \varphi_k$, $\lim \zeta_k(B^n) = x_0$. This contradicts the assumption that we have made. Consequently, $X \in \mathrm{ANE}\,(n)$.

Proposition 6.5. *If the mapping $f: X \to Y$ between the* $\mathrm{ANE}\,(1)$*-compacta stably preserves the* $\mathrm{AE}\,(n)$*-compacta for $n \ge 2$ and $I_i \in \mathfrak{S}(f)$ for $i < n$, then f is an $(n-1)$-soft mapping.*

Proof. By virtue of Proposition 6.4, the mapping f is $(n-2)$-soft and, consequently, open.

Assuming the contrary (f is not $(n-1)$-soft), we have $\{f^{-1}(y)\} \notin$ equi-LC^{n-2}, but $\{f^{-1}(y)\} \in$ equi-LC^{n-3}. This means that

(3) there exists a convergent sequence $y_k \to y_0 \in Y$;

(4) there exists $x_0 \in f^{-1}(y_0)$ and there exists a sequence of $(n-2)$-spheroids $\varphi_k: S^{n-2} \to f^{-1}(y_k)$ which converge to the point x_0 but do not admit of contracting the spheroids φ_k by $(n-1)$-films $\zeta_k: B^{n-1} \to f^{-1}(y_k)$ with the preservation of the convergence $\lim \zeta_k(B^{n-1}) = x_0$.

Since $Y \in$ LC^0, there are paths $s_k: [0,1] \to Y$, $s_k(0) = y_0$, $s_k(1) = y_k$, convergent to y_0. Since small homotopies of $(n-2)$-polyhedra are covered by small homotopies and $f^{-1}(y_0) \in$ AE(n), there exist $(n-1)$-films $\psi_k: B^{n-1} \to X$, $\psi_k \restriction S^{n-2} = \varphi_k$, $\psi_k(t \cdot \sigma) = s_k(t)$, where $0 \le t \le 1$, $\sigma \in S^{n-2}$, $\lim \psi_k(B^{n-1}) = x_0$. Clearly, $\psi_k(0) = x_0$.

Let us consider the monotone sequence $a_k \to 1$, $a_0 = 0$, on the interval $[0,1]$. It is easy to construct a mapping $s: [0,1] \to Y$ with the properties

(5) $s(a_{2k}) = y_0$, $s(a_{2k+1}) = y_{k+1}$ for all $k \ge 0$;

(6) the images of the intervals $[a_{2k}, a_{2k+1}]$ and $[a_{2k+1}, a_{2k+2}]$ under the mapping s coincide with the image Im(s_{k+1}) of the path s_{k+1}.

By the hypothesis, the fiberwise product $X_f \times_s [0,1] = (s^*)(X)$ is AE(n). Let $\chi_k: [-1,1] \to [0,1]$ be a piecewise-linear monotone mapping with $\chi_k(-1) = a_{2k-2}$, $\chi_k(0) = a_{2k-1}$, $\chi_k(1) = a_{2k}$. If we represent the sphere S^{n-1} as a suspension ΣS^{n-2}, then the relation $\xi_k: \Sigma S^{n-2} \to (s^*)(X)$, $\xi_k([t,\sigma]) = (\chi_k(t), \psi_k((1-t^2)\cdot\sigma)) \in (s^*)(X)$, where $-1 \le t \le 1$, $\sigma \in S^{n-2}$, correctly defines $(n-1)$-spheroids with the convergence property $\lim \xi_k(S^{n-1}) = (1, x) \in (s^*)(X)$. Since $(s^*)(X) \in$ AE(n), the spheroids ξ_k are contracted by n-films $\hat{\xi}_k: B^n = \Sigma B^{n-1} \to (s^*)(X)$, $\hat{\xi}_k \restriction S^{n-1} = \xi_k$, with the property $\lim \hat{\xi}_k(B^n) = (1, x_0)$.

Our immediate aim is the contraction of $(n-2)$-spheroids φ_k by small $(n-1)$-cycles lying in the fiber $f^{-1}(y_k)$. With the use of the following lemmas this must lead to the existence of small $(n-1)$-films that contract φ_k.

Lemma 6.6 [9, p. 78]. *Suppose that the compactum $X \in$ LC^{n-3} = AE$(n-2)$, $n \ge 4$, and the open sets $U \supset [V] \supset V \supset [W] \supset W$ are such that V is homotopically $(n-3)$-trivially contained in U and the composition $\psi: S^{n-2} \xrightarrow{\varphi} W \hookrightarrow V$ of $(n-2)$-spheroid φ_k and the embedding $W \subset V$ induces a zero homomorphism in Alexandroff-Čech $(n-1)$-homologies. Then φ is contracted by $(n-1)$-film $\hat{\psi}: B^{n-1} \to V$, $\hat{\psi} \restriction S^{n-2} = \psi$.*

Lemma 6.7. *If the compacta B and C are such that $B \cup C = B^n$, B contains the upper hemisphere $S^{n-1}_+ = \Sigma_+(S^{n-2})$ and C contains the lower hemisphere $S^{n-1}_- = \Sigma_-(S^{n-2})$, then the embedding $S^{n-1} \subset B \cap C$ induces a zero homomorphism in Alexandroff-Čech $(n-1)$-homologies.*

Let $B' = \{(x,t) \in (s^*)(X) \restriction t \le a_{2k+1}\}$, $C' = \{(x,t) \in (s^*)(X) \restriction t \ge a_{2k+1}\}$, $B = (\hat{\xi}_k)^{-1}(B') \subset \Sigma B^{n-1}$, $C = (\hat{\xi}_k)^{-1}(C') \subset \Sigma B^{n-1}$. We use Lemma 6.7: there exists an $(n-1)$-chain z on $B \cap C$ whose boundary ∂z is a generatrix $e \in H_{n-1}(S^{n-2})$. Then $z_1 = (\hat{\xi}_k)_*(z) \in H_{n-1}(f^{-1}(y_k))$ has a boundary $\partial z_1 = (\varphi_k)_*(e)$, i.e., z_1 bounds $(\varphi_k)_*(e)$. It is clear that $(n-1)$-chains z_1 are small as $k \to \infty$.

Lemma 6.6 implies the possibility of contraction of the $(n-2)$-spheroids $\varphi_k: S^{n-2} \to f^{-1}(y_k)$

by small $(n-1)$-films $\hat{\varphi}_k: B^{n-1} \to f^{-1}(y_k)$.

What is the condition that guarantees the belonging $I_n \in \mathfrak{S}(f)$, i.e., the covering of small homotopies by small ones? We write the answer to this question as an increasing sequence of classes of mappings between ANE(1)-compacta

$$\mathfrak{LS}_n \subset \{(n-1)\text{-Hurewicz bundles}\} \subset \{(n-1)\text{-Serre bundles}\}$$

$$\subset \{f \mid I_k \in \mathfrak{S}(f),\ k < n\}.$$

The last embedding is a finite-dimensional analog of the well-known theorem [14, p. 230]. Consequently, it remains to consider only the first inclusion.

Proposition 6.8. *If $f: X \to Y$ is a locally n-soft mapping, $n \geq 1$, then it is the Hurewicz $(n-1)$-bundle.*

Proof. Let us consider the many-valued mapping $\Phi: Y \to \mathfrak{F}(X)$, $\Phi(y) = f^{-1}(y)$, which is continuous in the Hausdorff metric ρ_H and has an equi-LC^{n-1}-family of images $\{\Phi(y) = f^{-1}(y)\}$. In this situation Michael's theorem [4] on the approximation of selections is valid: there exists a number $\beta > 0$ such that for any mapping $\varphi: Z \to Y$, $\dim Z \leq n$, and for any β-selection $r': Z \to X$ of the mapping $\Phi \circ \varphi$ (i.e., $r'(z) \in N(\Phi(\varphi(z)), \beta)$ for all $z \in Z$), whose restriction to the compactum $A \subset Z$ is a selection, there exists a selection $r: Z \to X$ of the mapping $\Phi \circ \varphi$ whose restriction to the compactum A coincides with r', $r \restriction A = r' \restriction A$. Suppose that Z is a compactum with $\dim Z < m$; and $g: Z \times \{0\} \to X$ is a partial lift of the homotopy $G: Z \times [0,1] \to X$. Using the number $\beta > 0$, we find a number $\delta > 0$ such that $\rho_H(f^{-1}(y), f^{-1}(y')) < \beta$ for all $\rho(y, y') < \delta$. Using $\delta > 0$, we find $\theta > 0$: $\rho(G(z,t), G(z,t')) < \delta$ for all $|t - t'| < \theta$ and $z \in Z$.

Let us consider the composition $\varphi: Z \times [0,\theta] \xrightarrow{\text{pr}} Z \times \{0\} \xrightarrow{g} X$ of the projection pr onto the factor Z and g. It is clear that φ is a β-selection for $\Phi \circ \varphi$. Since $\dim (Z \times [0,\theta]) \leq n$, there exists a selection $\tilde{G}: Z \times [0,\theta] \to X$ of the mapping $\Phi \circ \varphi$ which coincides with g on $Z \times \{0\}$. This means that $\tilde{G} \restriction Z \times \{0\} = g$, $f \circ \tilde{G} = G$. In the same way we construct a global lift $\tilde{G}: Z \times [0,\theta] \to X$ of the homotopy G.

To conclude the section, we shall give the proof of the theorem on the division of locally n-soft mappings by UV^n-mappings announced in [15].

Theorem 6.9. *Suppose that the locally $(n+1)$-soft mapping $h: A \to C$ can be decomposed into the composition $A \xrightarrow{g} B \xrightarrow{f} C$ of the* UV^n*-mapping g and the mapping f. Then f is a locally $(n+1)$-soft mapping.*

Proof. Since h is open, the mapping f is also open.

Next we note that all UV^n-mappings preserve the class of A[N]E$(n+1)$-compacta and all fibers of h are contained in this class. Therefore the fibers $f^{-1}(c) = g(h^{-1}(c) \in$ A[N]E$(n+1)$. Consequently, it remains to establish the equipotential property LC^n for the system of fibers $\{f^{-1}(c) \mid c \in C\}$.

For this purpose, we fix the point $b \in B$ and the number ε and seek $\delta > 0$, for which any partial mapping $B^{k+1} \hookleftarrow S^k \xrightarrow{\varphi} f^{-1}(c) \cap N(b_0, \delta)$, $k \leq n$, can be extended to the mapping $\hat{\varphi}: B^{k+1} \to f^{-1}(c) \cap N(b_0, \varepsilon)$. First of all, using ε, we find a number $\sigma = \sigma(e) > 0$ such that if $\rho(a, a') < \sigma$, then $\rho(g(a), g(a')) < \varepsilon/10$. Therefore, if $(\varphi, \psi) < \sigma$, then $(g \circ \varphi, g \circ \psi) < \varepsilon/10$.

Then we use the property equi-LC^n of the system of fibers $\{h^{-1}(c)\}$ and choose a number $v = v(s) > 0$ such that any mapping $e: P \to N(h^{-1}(c), v)$ of the $(n+1)$-dimensional polyhedron P into the v-neighborhood of any fiber h^{-1} can be approximated by the mapping $\tilde{e}: P \to h^{-1}(c)$, $(\tilde{e}, e) < \sigma/3$ into a fiber which coincides with e on the preimage $e^{-1}(h^{-1}(c))$ [16]. Thus, $\mathrm{Im}(e) \subset N(\mathrm{Im}(e), 2\sigma/3) \subset N(\mathrm{Im}(e), \sigma)$.

Since g is a mapping of compacta, there exists a number $v' > 0$ such that $\rho(b, b') < v'$ always implies $g^{-1}(b') \subset N(g^{-1}(b), v)$. Thus, if $\mathrm{diam}\, D < v'/2$ and $\rho(b, D) < v'/2$, then $g^{-1}(D) \subset N(g^{-1}(b), v)$.

Let us use the property of an approximative lifting of the UV^n-mappings g. According to this property, there exists a number $\beta = \beta(v') > 0$ such that any partial mapping $B^{k+1} \hookleftarrow S^k \xrightarrow{\chi} A$, $\mathrm{diam}\,(g \circ \chi) < \beta$ can be extended to the mapping $\hat{\chi}: B^{k+1} \to A$, $\hat{\chi} \restriction S^k = \chi$, $\mathrm{diam}\,(g \circ \hat{\chi}) < v'/6$.

And this is all. Now we take $\beta/4$ as $\delta > 0$. Suppose that the mapping $\varphi: S^k \to f^{-1}(c) \cap N(b_0, \delta)$ is defined. Since $g: g^{-1}f^{-1}(c) \to f^{-1}(c) \in \mathrm{UV}^n$ is a UV^n-mapping between ANE $(n+1)$-compacta, there exists a lift $\tilde{\varphi}: S^k \to g^{-1}f^{-1}(c) = h^{-1}(c)$ of the mapping φ such that $(g \circ \tilde{\varphi}, \varphi) < \beta/10$ and there exists $(\beta/10)$-homotopy H between φ and $g \circ \tilde{\varphi}$.

Since we have $\mathrm{diam}\,(g \circ \tilde{\varphi}) < \mathrm{diam}\,\varphi + 2(\beta/10) < \beta$ for the partial mapping $B^{k+1} \hookleftarrow S^k \xrightarrow{\tilde{\varphi}} A$, $\tilde{\varphi}$ can be extended to the mapping $\psi: B^{k+1} \to A$, $\psi \restriction S^k = \tilde{\varphi}$, $\mathrm{diam}\,(g \circ \psi) < v'/6$.

By virtue of the choice of v', we have $\mathrm{Im}\,\psi \subset N(g^{-1}f^{-1}(c), v) = N(h^{-1}(c), v)$. Therefore there exists an approximation $\tilde{\psi}: B^{k+1} \to h^{-1}(c)$, $(\psi, \tilde{\psi}) < \sigma/3$ and $\psi(b) = \tilde{\psi}(b)$ for all $b \in \psi^{-1}(h^{-1}(c))$. Since $\tilde{\varphi}(S^k) \subset h^{-1}(c)$, it follows that $\tilde{\psi} = \psi$ on S^k.

Since $\mathrm{diam}\,(g \circ \tilde{\psi}) \le \mathrm{diam}\, g \circ \psi + 2\varepsilon/10 < v'/6 + \varepsilon/10$, the image of $g \circ \tilde{\psi}: B^{k+1} \to f^{-1}(c)$ lies in the ε-neighborhood of the point b_0 (since $\delta + 2\beta/10 + v'/6 + \varepsilon/5 < \varepsilon$). The mapping $g \circ \tilde{\psi}$, together with the $(\beta/10)$-homotopy H, which connects $g \circ \tilde{\psi} \restriction S^k$ and φ, defines the required extension $\hat{\varphi}: B^{k+1} \to f^{-1}(c) \cap N(b_0, \varepsilon)$.

We denote by $\mathrm{M}(f)$ the cylinder for the mapping $f: X \to Y$ and by $d(f): \mathrm{M}(f) \to I$ the natural projection of $\mathrm{M}(f)$ onto the interval $I = [0, 1]$. It is known that this operation considerably improves the softness properties of mappings.

Proposition 6.10. *Let f be a mapping of* AE(m)*-compacta. Then f is a* UV^{m-1}*-mapping* $\iff$ *$d(f)$ is an m-soft mapping.*

Therefore, if f is a piecewise-linear mapping of AE(m)-polyhedra, then $d(f)$ becomes an m-soft piecewise-linear mapping. In all specific examples, this improves the softness properties up to the $(m + 1/2)$-softness. Thus, for instance, the natural simplicial UV^{m-1}-mapping $g: (S^m \times I) \cup (B^{m+1} \times \{0\}) \to \mathrm{Con}\, S^m$ generates, by means of the operation $d(g)$, half of the mapping $v_{m+1}: v_{m+1}^{-1}([0, 1]) \to [0, 1]$ which possesses stronger properties than m-softness. Therefore the following question seems to be interesting: does this improvement of properties occur automatically?

Question. Is it true that $d(f) \in \mathfrak{S}_{m+1/2}$ for any simplicial UV^m-mapping f of AE(m)-compacta?

A more general

Question. Is it true that the simplicial m-soft mapping f with AE(m)-fibers is $(m+1/2)$-soft?

7. APPLICATIONS OF THE DRANISHNIKOV RESOLUTION

Dranishnikov's resolution provides one more technique for constructing Edwards' resolution [18].

Proposition 7.1. *Any compactum X with the cohomological dimension $c-\dim_{Z} X \leq m$ can be covered by the* CE*-mapping $p: \widetilde{X} \to X$ with* $\dim \widetilde{X} \leq m$.

Proof. We represent the compactum X, lying in Q, with a natural convex metric, as the intersection $X = \cap P_i \times Q_i$ of the decreasing sequence of cylinders $P_i \times Q_i$, where the polyhedron P_i, defined in the triangulation τ_i, lies in the cube $\subset I^{N_i}$ and Q is the product of $\subset I^{N_i}$ and the Hilbert cube Q_i. Without loss of generality, we can assume that

(1) $\operatorname{mesh} \tau_i < \delta_i$, $\operatorname{diam} Q_i < \delta_i$, $3d_{i+1} < \delta_i < 2^{-i}$ and $N(x, 2\delta_i) \subset P_i \times Q_i$ for all $x \in X$.

We denote the preimage $(d_{m+1})^{-1}(X)$ by X^{m+1} and the preimage $(d_{m+1})^{-1}(P_{k+1} \times Q_{k+1})$ by N_k. Since $c-\dim_{Z} X \leq m$, the double width $a_m^{m+1}(X) = 0$. Consequently, for any k, the mapping $d_{m+1}: X^{m+1} \to X$ can be approximated by the mapping $f_k: X^{m+1} \to P_k^{(n)}$, $(d_{m+1}, f_k) < \delta_k$ (since $\dim X^{m+1} \leq (m+1)$). Since $P_k^{(n)} \in$ ANR, it follows that f can be extended to some neighborhood X^{m+1} in M^{m+1}. Without loss of generality, we can assume that f can be extended to the preimage $(d_{m+1})^{-1}(P_{k+1} \times Q_{k+1}) = N_k$ up to the mapping $f_k: N_k \to P_k^{(m)}$.

$$
\begin{array}{ccccccccccc}
 & X^{m+1} & \hookrightarrow & N_1 & & N_2 & \hookleftarrow & X^{m+1} & \hookrightarrow & N_2 & \dots \\
\swarrow f_1 & & \downarrow d_{m+1} & & s_2 \uparrow & \swarrow \bar{f}_2 & & \swarrow f_2 & \downarrow d_{m+1} & s_3 \uparrow & \dots \\
P_1^{(m)} & & X & & P_2^{(m+1)} & \hookleftarrow & P_2^{(m)} & & X & P_3^{(m+1)} & \dots
\end{array}
$$

The $(m+1)$-invertibility of d_{m+1} implies the existence of its section $s_k: P_k^{(m+1)} \to (d_{m+1})^{-1} \times (P_{k+1} \times Q_{k+1})$ over $P_k^{(m+1)}$.

We denote by $\widetilde{X}$ the limit of the inverse spectrum (S)

$$\varprojlim \{ \dots \xrightarrow{g_4} P_3^{(m)} \xrightarrow{g_3} P_2^{(m)} \xrightarrow{g_2} P_1^{(m)} \},$$

where g_k is the composition $P_k^{(m)} \hookrightarrow P_k^{(m+1)} \xrightarrow{s_k} (d_{m+1})^{-1}(P_k^{(m+1)}) \xrightarrow{f_{k-1}} P_{k-1}^{(m)}$. Since (S) consists of m-dimensional polyhedra, it follows that $\dim \widetilde{X} \leq m$.

Proposition 7.2. *If $\tilde{x} = (a_1, , a_2, a_3, \dots) \in \widetilde{X}$, $a_i \in P_i^{(m)}$, then there exists a limit $x = \lim(a_i)$ which belongs to X. Moreover, $x = \lim d_{m+1}(b_i)$, where $b_i = s_i(a_i)$.*

Proof. It is obvious that $f_i(b_i) = a_{i-1}$.

Therefore $\rho(a_i, a_{i-1}) = \rho(d_{m+1} b_i, a_{i-1}) = \rho(d_{m+1} b_i, f_{i-1} b_i) < \delta_{i-1} < 2^{-i}$. Consequently, there exist equal limits of the sequence $\{a_i\}$, $\{d_{m+1} b_i\}$, which belong to X by virtue of property (1) (see Proposition 7.1).

Thus, the mapping $p: \widetilde{X} \to X$, $p(\tilde{x}) = \lim a_i \in X$, is correctly defined and continuous. Let us show that the preimage $p^{-1}(x)$ of any point x is nonempty and coincides with the inverse limit of the spectrum (S_x)

$$\{ \dots \xrightarrow{g_4} R_3^{(m)} \xrightarrow{g_3} R_2^{(m)} \xrightarrow{g_2} R_1^{(m)} \},$$

where R_i is the intersection of the neighborhood $N(x, 2\delta_i)$ being contracted and the polyhedron $P_i^{(m)}$. It suffices to show that any point $\tilde{x} = \{a_i\} \in p^{-1}(x)$ is a thread of the spectrum (S_x). Indeed, it follows from Proposition 7.2 that $x = \lim a_i = \lim d_{m+1}(b_i)$. Therefore $\rho(a_i, x) \leq \rho(a_i, a_{i+1}) + \rho(a_{i+1}, a_{i+2}) + \ldots \leq \delta_i + \delta_{i+1} + \ldots < 2\delta_i$.

It follows from property (1) that R_i is nonempty, and, consequently, p is a surjective mapping.

The mapping $g_i\colon R_i \to R_{i-1}$ is the composition of the mappings $R_i \hookrightarrow T_i \xrightarrow{s_i} d_{m+1}^{-1}(N(x, \delta_{i-1})) \xrightarrow{f_{i-1}} R_{i-1}$, where T_i is the intersection of $N(x, 2\delta_i)$ with the polyhedron $P_i^{(m+1)}$. By virtue of the contractibility of the convex neighborhoods $N(x, 2\delta_i)$ and the general position theorem, the embedding $R_i \hookrightarrow T_i$ is homotopically trivial up to the dimension m. Therefore $g_i\colon R_i \to R_{i-1}$ also possesses this property. Consequently, $p^{-1}(x) = \varprojlim (S_x)$ is a UV$^\infty$-compactum, and p is a CE-mapping.

In the proof of Proposition 7.1 we can use any $(m+1)$-invertible mapping of an $(m+1)$-dimensional compactum on a Hilbert cube instead of the resolution d_{m+1}. The question arises whether we can use the complete collection of properties of the resolution d_{m+1} and prove a stronger result?

Problem (of an LC^m-resolution). Any one of the LC^m-compacta X of the cohomological dimension $\leq m$ can be covered by the LC^m-compactum $\widetilde{X}$ of dimension $\leq m$ by means of the CE-mapping $p\colon \widetilde{X} \to X$?

As another application of the resolution d_m we shall give the result concerning the Menger manifolds with the action of a group.

Proposition 7.3. *Suppose that G is a compact Lie group, $\mathbb{B}_\rho$ is a unit ball of the linear irreducible orthogonal representation ρ of the group G, $\mathbb{Q} = \prod \rho(\mathbb{B}_\rho)^\infty \times Q$ is an equivariant Hilbert cube. Then there exists a G-compactum $\mathbb{M}_m$ and a G-surjection $\delta_m\colon \mathbb{M}_m \to \mathbb{Q}$ with the following properties:*

(2) *δ_m is an equivariantly m-conservatively soft, and, consequently, it is polyhedrally G-m-soft, G-$(m, m-2)$-soft, and G-$(m-1)$-soft;*

(3) $\dim(\mathbb{M}_m/G) = m$, *δ_m is a strictly G-m-universal mapping;*

(4) *δ_m stably preserves G-AE(m)-compacta, in particular, $\mathbb{M}_m \in G$-AE(m).*

Proof. Let us consider the orbit projection $\pi\colon \mathbb{Q} \to \mathbb{Q}/G$ onto the space of orbits, which, as follows from Torunczyk's criterion, is the Hilbert cube Q. As the G-compactum $\mathbb{M}_m$ we take the fiberwise product $\mathbb{Q}_\pi \times_{d_m} M_m$ of the cube $\mathbb{Q}$ by the Menger compactum M_m relative to π and the Dranishnikov resolution $d_m\colon M_m \to Q$, as the G-mapping $\delta_m\colon \mathbb{M}_m \to \mathbb{Q}$ we take the projection which is parallel to d_m. The technique of constructing the G-compactum $\mathbb{M}_m$ and the G-mapping δ_m implies properties (2) and (3).

Since the resolution d_m is soft relative to the pairs H_i, $i \leq m$, the projection $\delta_m\colon \mathbb{M}_m \to \mathbb{Q}$, which is parallel to d_m, is also soft relative to H_i. For the same reason, $\delta_m \parallel d_m$ is a G-$(m-1)$-soft mapping and $\mathbb{M}_m \in G$-AE$(m-1)$.

The family of H-fixed points $\{\mathbb{Q}^H \mid H$ is a closed subgroup $G\}$ possesses the equi-LC^{m-1} property, and it is easy to verify that it is conservatively closed. By virtue of Proposition 4.3, this family passes into the family $\{(\mathbb{M}_m)^H = (\delta_m)^{-1}(\mathbb{Q}^H) \mid H$ is a closed subgroup of $G\}$ with the equi-LC^{m-1} property. From the characterization theorem [8] "$X \in G$-AE$(m) \iff \{X^H \mid H$ is a closed subgroup of $G\} \in$ equi-LC^{m-1} and $X \in G$-AE(0)" it follows that $\mathbb{M}_m \in G$-AE(m). We can establish by analogy the stable preservation by δ_m of G-AE(m)-compacta.

In the G-TOP category, the G-compactum $\mathbb{M}_m$ plays the part of an equivariant analog of Menger's universal compactum μ^m. Therefore a natural question arises as to the uniqueness of these objects.

Problem. Suppose that G is a compact Lie group, $\mathbb{X}$ and $\mathbb{Y}$ are G-AE(m)-compacts which have m-dimensional spaces of orbits $\mathbb{X}/G$ and $\mathbb{Y}/G$, and are also strictly G-m-universal. Is it true that $\mathbb{X}$ and $\mathbb{Y}$ are equimorphic?

We conclude this section by proving Theorem 1.5. We have to find the set $C \subset M_m$ which realizes the ample $(m-1)$-softness of the resolution d_m and the restriction of d_m on which is strongly m-universal in the class of Polish spaces. Proposition 2.3 that we proved earlier guarantees only the existence of dense G_δ-sets $C \subset \operatorname{Reg}_m d_m$. However, the general arguments are not sufficient for establishing the density of C in the fibers of d_m, and we must analyze the construction of the resolution in greater detail.

Recall that the Dranishnikov resolution d_m was constructed with the aid of the mappings $v_n: A_n \to [-1,1]$, $\pi_n: B_n \to [-1,1] \times S_n$, $p_{n+1}: D_{n+1} \to B^{n+1}$, $p^n_{n+1}: D^n_{n+1} \to K^{(n+1)}$, $f_m: D_m \to K$. Every successive mapping from this series was constructed with the aid of the operations of a fiberwise multiplication and the composition of mappings. Since it is easy to observe immediately the ample $(n-1)$-softness of the mappings v_n, π_n, p_{n+1}, p^n_{n+1} (say, $\operatorname{Reg}_n(v_n) = A_n \backslash \beta$), we can use Proposition 2.5 in order to establish the ample $(m-1)$-softness of $f_m: D_m \to K$. It is obvious that the mapping $\alpha: P \times Q \to K \times Q$ from Proposition 5.4 is also amply $(m-1)$-soft.

It is more difficult to establish the ample $(m-1)$-softness of the resolution d_m. Recall that the resolution was constructed as a limit mapping generated by the commutative diagrams (3_t) (see Sec. 5). It is easy to establish immediately the ample softness of these diagrams in the following sense.

Definition 7.4. The commutative diagram (4_t)

$$\begin{array}{ccc} X_{t+1} & \xrightarrow{\theta_t} & X_t \\ f_{t+1} \downarrow & & \downarrow f_t \\ Y_{t+1} & \xrightarrow{\sigma_t} & Y_t \end{array} \qquad (4_t)$$

is said to be *amply n-soft* if the characteristic mapping $g_t: X_{t+1} \to (Y_{t+1})_{\sigma_t} \times_{f_t} X_t$ of this diagram is amply n-soft.

We say that the set $C \subset X_{t+1}$ *realizes the ample n-softness of the diagram* (4_t) if $C \subset X_{t+1}$ realizes the ample n-softness of the mapping g_t (see Sec. 2).

In order to find the properties of ample n-softness when passing to the limit of inverse spectra, we shall prove the following important statement.

Proposition 7.5. *Suppose that the commutative diagrams* (4_t), $t \geq 1$, *generate the limit mapping* $f: \varprojlim\{X_t, \theta_t\} \to \varprojlim\{Y_t, \sigma_t\}$. *If, for all t, the diagrams* (4_t) *are amply n-soft, the mappings* σ_t *are* $(n+1)$*-soft, and the mapping* f_1 *is amply n-soft, then*

(5) *all the other mappings* f_t *and* σ_t *are also amply n-soft;*

(6) $\operatorname{Reg}_{n+1} f \supset \{(x_1, x_2, \ldots) \mid x_{t+1} \in \operatorname{Reg}_{n+1}(\theta_t), x_t \in \operatorname{Reg}_{n+1}(f_t)$ *for all* $t \geq 1\}$; *in addition, there exist* G_δ*-sets* $C_t \subset X_t$ *such that, for all* $t \geq 1$,

(7) C_t *realizes the ample n-softness of the mapping* f_t;

(8) $C_t = \theta_{C_{t+1}}$, $Y_t = f_t(C_t)$ *and the commutative diagram* (5_t^C)

$$\begin{array}{ccc} C_{t+1} & \xrightarrow{\theta_t \restriction} & C_t \\ f_{t+1}\restriction \downarrow & & \downarrow f_t \restriction \\ Y_{t+1} & \xrightarrow{\sigma_t} & Y_t \end{array} \qquad (5_t^C)$$

is $(n+1)$*-soft;*

(9) $C_{t+1} \subset \mathrm{Reg}_{n+1}(\theta_t)$, $C_t \subset \mathrm{Reg}_{n+1}(f_t)$.

Proof. Properties (5) and (6) follow from Propositions 2.4 and 2.6.

Let the sets $C_s \subset X_s$, which satisfy properties (7)–(9), be constructed for all $s \leq t$. We apply Proposition 2.6 to the fiberwise product $Z_t = (Y_{t+1})_{\sigma_t} \times_{f_t} X_t$ and find that

(10) the G_δ-set $\widetilde{C} = (\sigma_t')^{-1}(C_t)$ coincides with the fiberwise product $(Y_{t+1})_{\sigma_t} \times_{f_t \restriction} C_t$, lies in $\mathrm{Reg}_{n+1}(\sigma_t') \subset Z_t$, and realizes the ample n-softness of the mapping $f_t': Z_t \to Y_{t+1}$ (in particular, the set $\widetilde{C}$ lies in $\mathrm{Reg}_{n+1}(f_t')$ and is dense in the fibers f_t').

Suppose that the G_δ-set $X_{t+1}' \subset X_{t+1}$ realizes the ample n-softness of the characteristic mapping $g_t: X_{t+1} \to Z_t$ of the diagram (4_t). Then we take the intersection $X_{t+1}' \cap (g_t)^{-1}(\widetilde{C})$ as the set $C_{t+1} \subset X_{t+1}$. It is clear that C_{t+1} realizes the ample n-softness of the mapping $g_t \restriction (g_t)^{-1}(\widetilde{C})$, and, consequently, the commutative diagram (5_t^C) is $(n+1)$-soft. This completes the proof of property (8).

By virtue of (10), $C_{t+1} \subset (g_t)^{-1}(\mathrm{Reg}_{n+1} f_t' \cap \mathrm{Reg}_{n+1} \sigma_t') \cap \mathrm{Reg}_{n+1} g_t$. Then property (9) easily follows from Proposition 2.2 (on the composition of kernels). It remains to establish the density of C_{t+1} in the fibers of the mapping f_{t+1}. But this is the fact since $f_{t+1} \restriction C_{t+1} = f_t' \circ g_t \restriction C_{t+1}$, the set $\widetilde{C}$ is dense in the fibers of the mapping f_t', and X_{t+1}' is dense in the fibers of the mapping g_t.

Let us now establish the ample $(m-1)$-softness of the Dranishnikov resolution d_m. We apply Proposition 7.5 to the amply $(m-1)$-soft diagrams (3_t) and obtain the commutative diagrams (6_t) which satisfy properties (7)–(9),

$$\begin{array}{ccc} C_{t+1} & \xrightarrow{\theta_t \restriction} & C_t \\ e_{t+1}\restriction \downarrow & & \downarrow e_t \restriction \\ I^{t+1} & \xrightarrow{\sigma_t} & I^t \end{array} \qquad (6_t)$$

They generate the mapping $e: C = \varprojlim\{C_t, \theta_t \restriction\} \to \varprojlim\{I^t, \sigma_t\}$ which coincides with the restriction $d_m \restriction C$. Since C_t was a dense G_δ-subset, according to the Baire theorem on the category, C is also a dense G_δ-subset in X. Since the embedding

$$\mathrm{Reg}_m(d_m) \supset \{(x_1, x_2, \ldots) \mid x_{t+1} \in \mathrm{Reg}_m(\theta_t), x_t \in \mathrm{Reg}_m(e_t)\}$$

holds by virtue of (6), the set C lies in $\mathrm{Reg}_m(d_m)$.

Since C_t realizes the ample $(m-1)$-softness of e_t and the diagrams (6_t) are m-soft, C is dense in the fibers of the resolution d_m.

Before proving the strong m-universality of the restriction $d_m \restriction C$ in the class of *Polish* spaces (i.e., in the class of G_δ-subsets of the Hilbert cube), we shall give the necessary definitions.

Definition 7.6. The mapping $f: X \to Y$ between Polish spaces is said to be

(a) *m-complete* if, for any mapping $\psi: Z \to Y$ of the m-dimensional Polish space Z, there exists a closed embedding $\varphi: Z \to X$ for which $f \circ \varphi = \psi$;

(b) *strongly m-universal* if, for any open covering $\omega \in \operatorname{cov} X$ and for any mapping $g: Z \to X$ of the m-dimensional Polish space Z, there exists a closed embedding $h: Z \to X$, which is ω-close to g, such that $f \circ g = f \circ h$.

Proposition 7.7. *If all the spaces in m-soft commutative diagrams* (5_t^C), $t = 1, 2, \ldots$, *are Polish, all mappings are m-soft, and the characteristic mappings* $g_t: C_{t+1} \to Z_t$ *of these diagrams are m-complete, then the limit mapping* $f \upharpoonright C$ *is strongly m-universal.*

This fact can be establish by analogy [5, Lemma 3]. Since the commutative diagrams satisfy the conditions of this proposition, the proof of Theorem 1.5 is complete.

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